

# Mathematical Finance, and The Rise of the Modern Financial Marketplace

Assad Ebrahim\*

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## Abstract

One of the fascinating areas to arise recently in applied mathematics has been Mathematical Finance. From a technical point of view, Mathematical Finance uses a broad range of sophisticated mathematics for its financial models, and relies on state-of-the-art software engineering and computer hardware to implement these financial models, often in real-time.

Whether one is interested in technology or not, there are a kernel of core financial ideas at the heart of the global free market capitalist system that every literate citizen should understand. Whether we agree with their principles or with the inequities that are, arguably, the result, these ideas are in place across most of the world today. A closer look at mathematical finance will offer a better understanding of the mechanics of the modern financial world.

In this article, I'll motivate the need for financial mathematics through a simplified account of the rise of the modern financial marketplace.

## What is Mathematical Finance?

**Introduction** One of the fascinating areas to arise recently in applied mathematics has been Mathematical Finance. This is a field whose development has occurred largely within the past forty years, with explosive growth taking place over the past twenty years.

From a technical point of view, Mathematical Finance uses a broad range of sophisticated mathematics for its financial models: from the partial differential equations of mathematical physics, to stochastic calculus, probabilistic modeling, mathematical optimization, statistics, and numerical methods.

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\*Assad Ebrahim is an Applied Mathematician, Systems Engineer, and Technical Director. He has led the design and development of a number of real-time sonar and software systems for the detection, analysis and classification of underwater phenomena using acoustics, with applications in environmental monitoring, undersea defense, and off-shore aquaculture. Formerly the Director of Engineering at BioSonics, Inc. (<http://www.biosonicsinc.com>) in Seattle, U.S.A., Assad is now a Consulting Systems Engineer in the UK, and works with marine technology and intelligent systems companies on developing advanced underwater systems (<http://www.goscience.co.uk>). He is also the founder of Mathematical Science & Technologies (<http://www.mathscitech.org>), an organization involved in the application of mathematics, technology, and applied science to industry, and in the enhancement of curricula for STEM (Science, Technology, Engineering, and Mathematics) education. Assad received his M.Sc. in Applied Mathematics from the University of Washington in Seattle and his B.A. in Mathematics from Swarthmore College in Philadelphia. Contact Information: [assad.ebrahim@mathscitech.org](mailto:assad.ebrahim@mathscitech.org).

But mathematics is just one of a two-headed technology requirement. The practical implementation of trading strategies based upon these mathematical models requires designing efficient algorithms as well as exploiting the state-of-the-art in software engineering (real-time and embedded development, low latency network programming) and in computing hardware (FPGAs, GPUs, and parallel and distributed processing).

Together the technical aspects of mathematical finance and financial engineering lie at the intersection of business, economics, mathematics, computer science, physics, and electrical engineering.

Whether one is interested in technology or not, there is a kernel of core financial ideas that every literate citizen should understand. These ideas are at the heart of the global free market capitalist system that, for better or worse, is in place across most of the world today. Whether we agree with its principles, its structures, or the inequities that are arguably exacerbated as a result, a closer look at mathematical finance offers a better understanding of the mechanics of the modern financial world. For the technologically inclined, there are ample opportunities to contribute.

**What is Mathematical Finance?** Mathematical Finance, also called Quantitative Finance, is that branch of applied mathematics that is applicable to the needs of financial markets. Mathematical Finance develops and extends the models of financial behavior that are suggested by financial economics. Since the future is inherently unknown, mathematical finance is a data-intensive, stochastic subject and relies on simulations for many of its modelling formulations, for computing expected values of investment portfolios, and for evaluating and enhancing its trading strategies. Those who work in this field are called “quants”, an abbreviation for quantitative analyst, or financial analyst.

Financial Engineering, also called Computational Finance, is focused on the practical implementation of the models of mathematical finance and the simulations necessary to evaluate trading strategies. The mission is the engineering design and development of platforms and systems, often real-time, that are able to take in financial data and rapidly perform the large scale numerical simulations (monte-carlo, simulated annealing, etc.), for computing the likelihood of various outcomes and exploring refinements toward a preferably risk-minimizing strategy.

How did financial mathematics arise and why is it needed? In what follows, I'll motivate the need for financial mathematics through a simplified account of the rise of the modern financial marketplace.

## **A Simplified Account of the Rise of the Modern Financial Marketplace**

For all its present complexity, the evolution to the modern financial marketplace appears quite natural when viewed as the response to three basic questions that have been asked, over time, by increasingly more powerful actors, and involving increasing larger sums of capital. The three basic questions are:

1. I have excess capital (cash) – where can I put it safely?
2. I need capital – where can I get it reliably?

3. I want to improve upon my present capital position and reduce the exposure of my capital to risk – how can I do this?

Let's take a look, moving from early history to more modern times. To keep the narrative light, I'm going to assume for the first part that you're the one with excess capital. In the second part, I'll take up the perspective from the point of view of a banking house in the pre-modern era of emerging statehood, national interests, expanded commerce and larger standing armies.

Along the way, observe how the increasing complexity of possible financial transactions leads to increasingly complicated questions of valuation, risk, terms, discounts, offers, and profit margins. These are the calculations that, carried forward to modern times and the modern financial landscape, form the subject of mathematical finance.

## **From Excess Capital and Mattresses to Money-lending and Banking Houses**

**Stage 1: Keeping Excess Capital in your Mattress or in a Hole** You have excess capital. Where do you put it? You could keep it in your mattress. There, it will earn no interest. And there is the attendant risk of theft, fire, rats, moisture and other ways in which your capital can be diminished.

Why? Because you are a little guy. Living in a little home. Vulnerable to being bullied, robbed, and ransacked. If it were known that you had a lot of cash in your mattress, you can be fairly sure of the outcome. You would undoubtedly have the pleasure of marauders whose perception of the risk of harm from you or chance of being caught by the authorities is less than their expectation of the profitability of breaking down your bedchamber door and un-stuffing your mattress.

Burying your capital in a hole in the ground is only slightly better. In effect, a hole in the ground is a fee-free deposit vault, but unfortunately the combination is simply the location. If at best, no one sees you make your deposit and you tell no one, your funds are safe assuming no accidental discovery – wind, earthquake, a dog, or someone digging another hole in the same place. At worst, your deposit is seen or you are followed, and your capital is dug up soon after you disappear around the corner.

Even assuming that you are able to dig, bury, and cover your deposit in absolute secrecy, there is another downside: once deposited you no longer have ready access to your funds. Every access to your funds requires unearthing and reburial, and any failure of secrecy in the process can mean the disappearance of the remaining funds.

So, where do you keep your money safe against unexpected impoverishment but also get reasonably reliable, safe access to your funds?

**Stage 2: Armed Security: A “Safe Mattress” For a Fee** In such circumstances, you might decide to arm yourself so that you can personally protect your small fortune. But this puts at risk the one thing that is required in order to enjoy your wealth – your life.

So, it is reasonable to consider placing your money into the care of an agent whose business is the safe keeping and defense of many people's money, not just yours. They would have to be well armed and well protected in order to provide a vigorous defense of the physical security of your money. You would, of course, have to trust the agent, and they would have to be worthy of that

trust, otherwise you have just watched your money run away under an armed guard of your own choosing. And ideally, you would like to be able to withdraw all or part of your money readily at any time.

Now, armed deposit takers who would offer these services of safekeeping and ready access to deposited money, would do so for a fee since the services involve no small risk of attack, and require arms and guards to address this risk. But since this is a more appealing choice than either a hole in the ground (access problems) or your mattress (security problems), there would be those more than happy to pay a fee for the service.

In this way, those with substantial excess wealth either become their own fortified depositories, or pay a fee for that service, leading to the development of professional money-holders.

**Stage 3: From Money-holders to Moneylenders to Banks** Switching perspectives to the point of view of our armed agent: over time, he has become a well known and trusted money-holder who, for a fee, holds sums from an increasing number of members of the community, amassing an increasing volume of physical wealth. What does the money-holder do with all the money that is deposited with him?<sup>1</sup>

In all likelihood, those on sudden hard times needing a short infusion of capital, would have already come to the money-holder to inquire about the possibility of obtaining a loan. Under the pressure of their own circumstances, they are likely to be willing to pledge an additional fee (interest) in addition to repaying the principal, for the benefit of being able to borrow.

But before making loans with deposited capital, the money-holder would have to consider carefully: firstly, the risk of a default by the borrower. In the event of a default, the money-holder would himself have to make whole the entire amount to repay the original depositor's deposit, himself bearing the loss from default.

What does the money-holder do? If he is ambitious and risk-taking, he charges interest for loans and begins to function as a (non-insured) bank. On the one hand, he accepts deposits for a deposit fee and pledges to hold your money safely for you until such time as you want it back. But behind closed doors, he loans out your money to borrowers, and charges them a lending fee (interest). (We'll discuss interest in more detail later.)

As this goes on, the money-holder/moneylender must consider a second risk and must decide how much of the deposited capital should be held in reserve and how much should be allowed to be let out the back door as loans to others. The risk from lending too much is the situation of not having enough liquidity to return a deposited amount when a depositor attempts a withdrawal. This situation becomes extreme when war, rumors, fear, unrest, or other unstable circumstances cause a large number of depositors to rush to withdraw their money in a short space of time.

However, apart from the risk of having insufficient liquidity to cover withdrawals, all other self-interested considerations strongly support the money-holder lending a portion of deposited capital. By doing so, he increases his

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<sup>1</sup>He could bury it in a hole, but this has the same disadvantages associated with an individual, only on a larger scale. Certainly, from the fees levied for this service, the money-holder would fortify his home and employ a larger group of better armed men to guard against attack. But if all of the capital resides in one location, even a well-guarded fort becomes an increasingly attractive target for coordinated assault, or treachery from within. Over time, simply holding money in one location becomes an increasing burden.

profits. Not only is he taking fees from depositors, but he is collecting regular interest on loaning out the deposited capital, which otherwise would sit idle. The additional profits from interest can be structured so as to cover expected losses from defaulting borrowers, or those from whom collection attempts fail. Finally, in the event of attack, his loss exposure is reduced since the deposited assets are dispersed among borrowers.

From excess capital to money-holders, and from money-holders to moneylenders, we see individuals that formerly acted as holders of money or that had reserves of excess capital, begin to make loans and collect interest on these while continuing to charge fees to depositors to hold their money safely and make it available on demand. Thus, the origin of banking houses.

**Why Ordinary Citizens Do Not Easily Become Moneylenders** If this is indeed a natural pathway, one might expect larger numbers of ordinary citizens with a sufficient amount of excess capital attempting to directly invest by making loans and collecting interest.

But doing so profitably requires an increasingly specialized infrastructure. There is the requirement of maintaining a sufficiently strong armed force to protect your capital and your person. You would also need goons to collect the debts and payments from borrowers reluctant to meet their obligations. You would need a “counter”, or store front, to which depositors could come to withdraw or add funds, and to which prospective borrowers can come to request loans. Finally, you would need an accounting function to keep track of the payouts and receipts, as well as the profitability of the operation.

The two functions of rudimentary banking houses, money-holding and money-lending, have become a specialized business of their own, and it is this specialization that limits the number of participants.

## **How a Stronger State Provides Further Impetus to Finance**

The evolution of simpler money-lending and collection, to the more complex systems of contractual, asset backed loans accompanies the maturation of the nation state and its attendant institution: the rule of law, courts for claimants, the honoring of contracts as a state value, and state-backed enforcement.

All of these developments contribute to the maintenance of good order in financial transactions. The resulting financial stability benefits banking houses as well as all those engaged in capitalist activity: borrowers, lenders, merchants, speculators.

**The Evolution of Lending** As time goes on, banking houses that hold money and lend money become an essential part of the fabric of personal and commercial life. With increasing accumulated reserves of capital among banking houses, and increasing familiarity and patronage by the affluent and ambitious of the city, banking houses now begin to compete with each other to try to attract depositors and borrowers, knowing that they earn fees from both and require both in order to continue their profitable business.

One area of competition among them is the terms for deposits and loans. For depositors willing to fix their deposit for a longer period, the banking house may offer a discounted deposit fee as incentive. This provides long term reliable liquidity – good news for lending.

For loans, there may be different fees for different types of loans: unsecured loans (riskier) and secured loans (safer). Whereas defaulting on a loan

in earlier times often meant enslavement or indentured servitude, the onset of state enforced debt protection laws limited the recovery of unsecured loans to the pittance that a debtor's prison might afford – effectively these loans were write-offs.

Secured loans on the other hand, are backed by the title deed to an asset which is then held by the lending agent in case of the borrower's default on the loan. Borrowers who secure their loan with an asset that they own reduce the lender's risk of loss from default, since in such a case the lender gets the asset. A lending house being asked for a secured loan might therefore charge less interest to such clients.

Profiting from secured loans, however, requires that the lender (banking house) be able to accurately assess the value of the asset against the eventuality of having to dispose of it at some point in the future in order to recover the outstanding amount of the loan on which a borrower might default.

**The Role of the State** Notice that evolution of the financial marketplace to one offering diversified loan products requires a vital change, brought about by the increased strength of the state. Most importantly, the notion of a secured loan requires a stronger notion of private property, of contracts, and an arena for the enforcement of contracts that is more than simply a contest of strength between debt collector and delinquent. Of course, a stronger state and stronger voices of the people also mean additional limitations. For example, the abolition of penal consequences for debt default makes unsecured loans increasingly risky from the lender's point of view of guaranteed profit. But rather than stifle the market for these loans, banking houses have responded to these and other constraints by factoring these into their risk and profit calculations.

## **The Rise of the Modern Financial Marketplace**

With increasing confidence in the enforcement apparatus of the state and the reliability of contractual obligations, the stage is set for money to be borrowed by those offering incentives other than the typical interest or asset backed security. We then have venture capital, bonds, stocks, securities, and insurance.

And every diversification in the borrowing and lending business increases the incentive to develop better means of evaluating risks, assessing non-cash value, and forecasting valuations into the future, strengthening the role of financial analysis in the profit engine of the banking house.

**From Moneylender to Financier (Venture Capital) and Investor (Bonds and Stocks)** One opportunity open to those with excess capital is the partial financing of a venture. A banking house or moneylender provides part of the capital in return for a share of the expected profits. The venture might be a trading expedition that requires capital to buy goods, a ship, and to pay a crew. A successful trading voyage promises to return with locally scarce items (spices, tea, woods, furs, metals) that will fetch a premium and allow the trader to repay the investment plus pay out a portion of the profits and still make out handsomely for himself and possibly the crew (if their labor was taken as an investment instead of for wages).

A less direct investment is the buying of a bond. A bond is issued by an individual or organization looking for capital and willing to repay the capital with interest. The only way to lose on a bond is if the bond-issuer defaults.

In which case you will obtain a portion of the proceeds from a fire-sale of their assets.

Still less direct is the buying of a stock. A stock is issued by a company looking for capital and willing to pay out a dividend (share of the profits). Because a company that is doing well will be increasing likely to pay out dividends, their stock price rises. However, there is a risk that the opposite can happen, in which case the stock holder may lose all or part of their initial investment. But the fact that selling stock at the right time can yield substantial profits means that the stock market becomes attractive to those interested in relatively short-term speculation on the fortunes of businesses and industries.

In all three situations (venture funding, bonds, stocks), banking houses, through the 1700s and 1800s provided the crucial capital responsible for significant expansion of economic activity through overseas exploration, colonization, business expansion, infrastructure development, indeed, the bankrolling of governments, and the funding of warfare and standing armies. In return for their capital, they negotiated either some form of ownership in the venture or entitlement to a share of the profits. The arrangement meant profit for them, and liquidity for merchants, traders, adventurers, speculators, and governments – a Faustian bargain for some, but a source for dramatic economic benefits for others.

**From Money-holder to Insurer: The Rise of Insurance** With the increasing complexity of the financial landscape and various contracts, loans, transactions, and investments, the desire increases for different kinds of insurance to offer protection against different loss scenarios: life, health, value, assets, business, fire, flood, earthquake, travel, default, a drop in value either through accident, attack, damage, loss, or insolvency.

The desire for insurance is obvious: one perceives a risk to one's wealth or investment and wishes to protect one's assets against diminishment due to unpredictable losses of various kinds. Clearly one would expect to have to pay a fee to the insurer for taking the risk of having to make whole the insured value. And, over time, these fees would diminish your capital. But, in the event of a catastrophe, you would at least not lose the entire asset.

From the point of view of the insurer, insurance is another business opportunity for those with excess capital. For a fee, one could insure the assets of others. Success in the business of insurance, even more so than the loan business, is directly related to the ability to accurately assess risk, likelihoods of various outcomes, and to charge appropriate premiums to allow the business to be profitable. The mathematics of profitable insurance pricing is actuarial science.

**The Sovereign as a Borrower in the Financial Market** Perhaps the largest player in the evolution of finance to its modern day situation has been the entry of the sovereign himself. What institution can offer the greatest security against default? Historically, it was the sovereign himself, later the state, or government. But why does the government require liquidity? Typically it has been the desire for or threat of war, and the rapidly increasing cost of standing armies and modern warfare in general, that has forced the sovereign to seek additional liquidity beyond its natural base of income. Other reasons have been the construction of heavy infrastructure: roads, bridges, ports, airports, stadiums – all of which require cash. Or perhaps its treasuries are low and

inadequate to cover the day-to-day needs of the state.

The sovereign or state typically has a good credit rating, i.e. is trusted to repay the loan. They have wealthy citizens, flourishing businesses, farms, etc., from all of which they collect taxes, a significant source of national revenue. So the sovereign issues a bond: that is, a contract that can be purchased. The contract is a pledge by the government to repay the face value of the bond at a fixed time in the future (maturity) and with a fixed interest rate.

In this manner, the rise of big government, nation states, and the exponentially increasing cost of warfare, of raising and financing standing armies, of building infrastructure, maintaining overseas holdings, all expanded the needs, volumes, and clients of financial activity to an inter-state level, including the government issuing of bonds.

Just as stocks did, bonds also become something that can be traded. They become assets with a face-value worth plus additional interest to be paid on them, and a (typically small) risk of default. By brokering the buying and selling of bonds, the financial markets have made lending an activity that everyone can participate in – small amounts from many people – thus the democratization, if you will, of money-holding and money-lending “to the masses”.

**Expanded Mercantilism as a Catalyst for Financial Activity** Coincident with the rise of nations was the dramatic expansion of the middle class and the development of extensive and regular commerce and trade internationally, along with expeditions of exploration, conquest, and colonial exploitation. These activities also required capital, often on credit, often as investments, venture capital, or stock.

The accompanying proliferation of financial activity led to the creation of trading exchanges, brokers, insurers and re-insurers. Larger streams of cash flow, easier capital, all led to the rise and expansion of commodities markets, stock markets, bond markets, and various specialized exchanges for the trading of contracts, securities, and investment vehicles.

The various elements of the modern financial landscape were thus born: capitalists (those with excess capital), banks, financiers (banks, wealthy individuals), insurers and re-insurers, brokers, and exchanges becoming fixtures in the financial life of modern cities.

**From Financial Analysis to Mathematical Finance: Investment, Valuation, and Risk Assessment** As the transactions and arrangements of capitalists and financiers (banks, wealthy individuals, moneylenders, insurers) became increasingly complex, there arose the need for better ways to compute the optimal strategies for assessing fees, generating profits, minimizing losses, forecasting changes in value, and developing profitable investment strategies.

Mathematics has always been involved, but in the past thirty years, the application of techniques of advanced mathematics have transformed both the ability to more accurately model financial behavior, and to manage risk more analytically. The collection and refinement of the techniques of valuation, risk assessment and investment strategy are at the heart of mathematical finance.