The Prehistoric Origins of Mathematics

By **Assad Ebrahim**, on November 5th, 2019 (19,821 views)

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How far back in time can we trace mathematical practice? Finding an answer will take us on a journey across multiple disciplines, including discoveries made within the last 30 years in archaeology, Assyriology, artifact analysis (close reading theory), anthropology, linguistics, genomics, neurobiology, and animal cognition.



Cover illustration from The Sumerian King List (Source: **Jacobsen/1939**)

Summary

Here's a summary of what we know, as a roadmap to where we're going:

- By 3,200 BCE (5200 years ago) there is indubitable evidence for mathematical practice in Sumerian city states using clay tokens and "bullae" (clay envelopes) for book-keeping within a scribal-statal administrative context. This was the earliest known system of accounting, using fixed values for specific commodities, and the earliest known form of proto-writing. (cf. **Section 1**)
- Between 6,000 BCE and 4,000 BCE (8000 to 6,000 years ago) there is evidence of (1) painted pottery showing elaborate designs using sophisticated symmetries, and (2) layout of prestige buildings (eg temples, shrines) showing architectural competency in geometric design (parallel lines, application of right triangle ratios such as 3:4:5 and 5:12:13, use of squares and rectangles) and precise build execution with support for use of a standardized length measure. (cf. **Section 2**)
- Excluded from this list are a handful of artifacts dating from 18,000-70,000 BCE (Ishango, Lebombo, and Wolf bones and engraved ochre from Blombos Cave) because evidence for the mathematical nature of their markings is based on controversial close readings by Marschak, and would require ignoring reasonable non-mathematical alternative hypotheses. (I am reminded of Newton's famous epistemological maxim published in 1713, "**Hypothesis non fingo"** meaning "I need feign no hypotheses." [Walsh/2010] (cf. **Section 3**)

At this point, direct evidence dries up, so we consider indirect evidence with a revised question:

When did the capability emerge for mathematical thinking (number, shape, time, change)?

- Around 250,000 years ago, the archaeological record shows changes in human species as anatomically modern humans diverged from Homo Erectus. Evidence of complex behaviour (ritual burial of the dead, cooperative hunting, the controlled use of fire, language capability) suggests the *capacity* for symbolical

thinking that would be a prerequisite for any sort of mathematical practice (counting, bijection, keeping a tally, measuring, symmetry, or abstract artistic design) (cf. **Section 4**)

- Around 2.4 million years ago, the fossil record shows the first appearance of technologically complex stone knapping tool-making by earlier human species (Australopithecus or Homo Erectus) around Lake Turkana, in a localized acceleration beyond the simple split-stone tools of the prior Oldowan culture (this is 800,000 years before the Acheulean culture with comparable technology). (cf. **Section 5**). These findings, along with the recent discovery of the Piraha people of the Amazon (whose language has no evidence of any numeracy) support the gradual conceptual progression of human-kind from index/response (shared with animals) to iconic thinking far earlier than previously supposed, and on the way toward symbolical thought (cf. the semiotic model, **Section 6**)
- Around 150 million years ago neurological and biological evolution in reptiles (last common ancestor of birds and mammals) had progressed far enough to allow index/response mechanisms that would qualify as “number sense”, though obviously not mathematical practice in any iconographic or symbolical sense. (cf. **Section 7**)

A list of recommended readings, most of which can be downloaded freely, is provided in the **Bibliography**.

A set of **Appendices** provide additional color on specific topics: the dialectic nature of arithmetic (and mathematics) (**Appx 1**), the origin of writing (**Appx 2**), summary of Mesopotamian cultural history (**Appx 3**), how mesopotamian written historical events are dated (**Appx 4**), what life was like in the transition to the neolithic period (**Appx 5**), timeline of the stone age (**Appx 6**), and birth of the universe to the origin of man (**Appx 7**),

1. Clay token systems for counting and measurement in Sumeria (Uruk and Susa): evidence from 3,200 BCE (5200 years ago)

The strongest archaeological evidence of mathematical practice dates to at least 3,200 BCE (5200 years ago) in ancient near eastern city-states. Archaeological finds in the past century have shown that geometrical clay tokens which appear to have been used for counting and measuring across the region, became established at this time in the Sumerian city-state of Uruk (southern Mesopotamia/Iraq) as the standard administrative procedure for recording commercial transactions (archaic book-keeping). Similar finds have been made in Elamite Susa (Zagros mountains/western Iran), a rival city-state to Uruk. [**NissenDE/1993**], [**Friberg/1984**].



Near East ca 4300 BCE toward the end of the Ubaid period and before the earliest discovery of accounting. Notice the many city-states that had arisen in the alluvial flood plains between the Tigris and Euphrates rivers. In particular, Uruk and Susa would rise as prominent city-states. (Source: **Wikipedia**)

Key to this conclusion were the finds by Denise Schmandt-Besserat of tokens enclosed in clay “bullae”, or sealed clay envelopes, with matching token-impressions on the clay surface. ([**Besserat/1977**], [**Oppenheim/1959**]) The impressed indentations made by pushing the tokens into the wet clay are also the earliest examples of proto-writing. [**Damerow/1999w**]

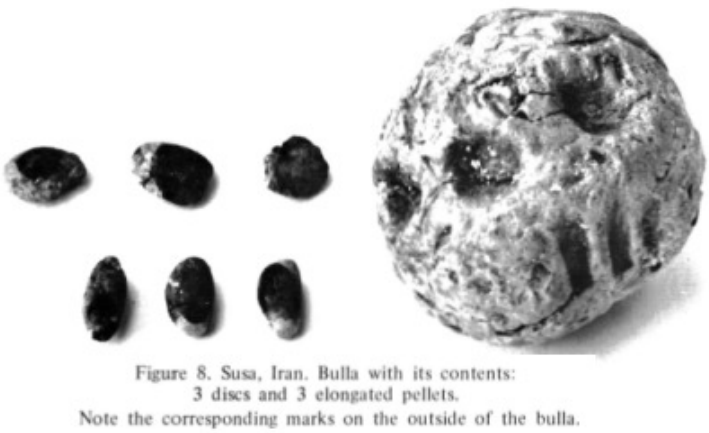


Figure 8. Susa, Iran. Bulla with its contents:
3 discs and 3 elongated pellets.
Note the corresponding marks on the outside of the bulla.

Clay tokens and bulla (envelope). Note the impressions of the tokens on the surface of the bulla (Source: **Besserat/1977**, SMS 1, p53)

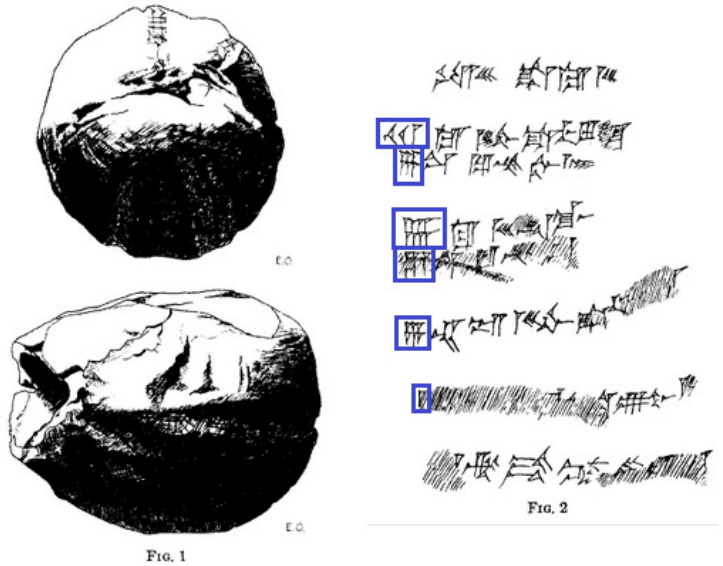
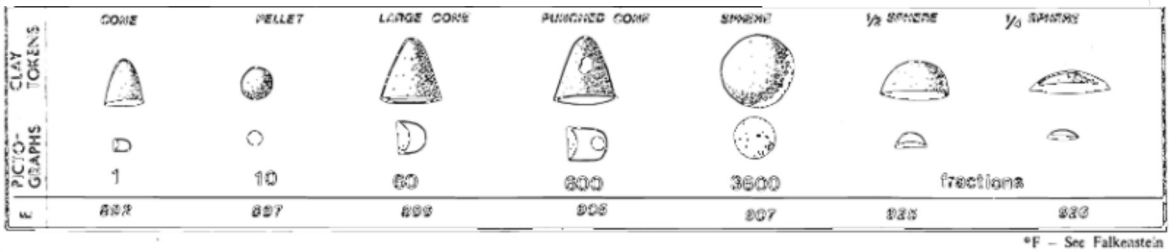


Fig. 1

Fig. 2

Bulla with 48 tokens found at Nuzi site dated from 2000-1500 BCE after cuneiform writing was fully developed, with an inscription describing the meaning of the tokens inside. (Source: **Oppenheim/1959**)



*F – See Falkenstein

Clay tokens mapped to the pictographs and numerical values assigned to them. (Source: **Besserat/1977**]

The meaning of these impressions can be worked out from the historical sequence of clay tablets that initially record token-impressions only with no additional written context, to later the juxtaposition of number signs with additional signs indicating the commodity (e.g. 3 sheep), and finally the use of separate **cuneiform** signs for number and for commodity. [**Nissen/1986**], [**NissenDE/1993**], [**Robson/2000**].

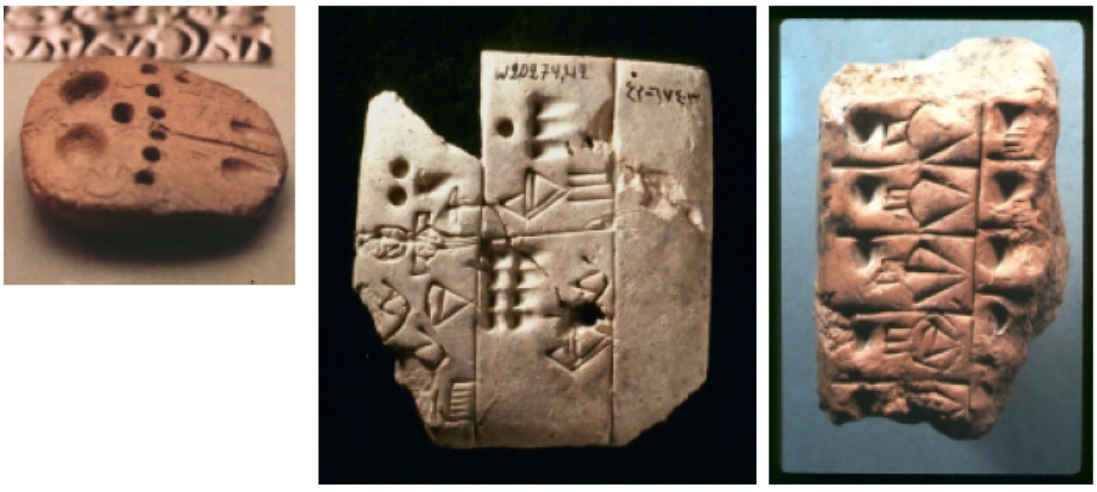


Figure 2. Hierarchically structured proto-cuneiform administrative document (W 20274,42 with an entry and two subentries) and „list" of vessels in a school text (fragment W 24157 of the vessels list, see Englund and Nissen, 1993). Both texts are depicted in conventional orientation turned 90° to the left.

(left) simple clay tablet with token impressions for quantity (commodity is presumably implicitly known by context). The shape of the token impressions led to the early metrological signs. (middle & right) complex numerical tablets showing quantity and commodity separately. (Source: **Damerow/1999w**)

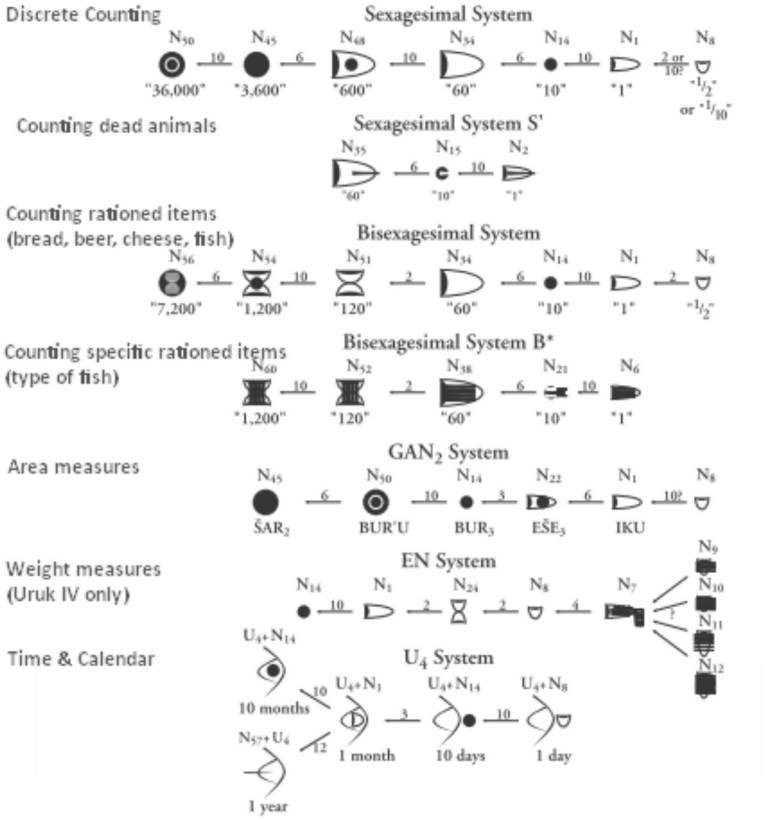
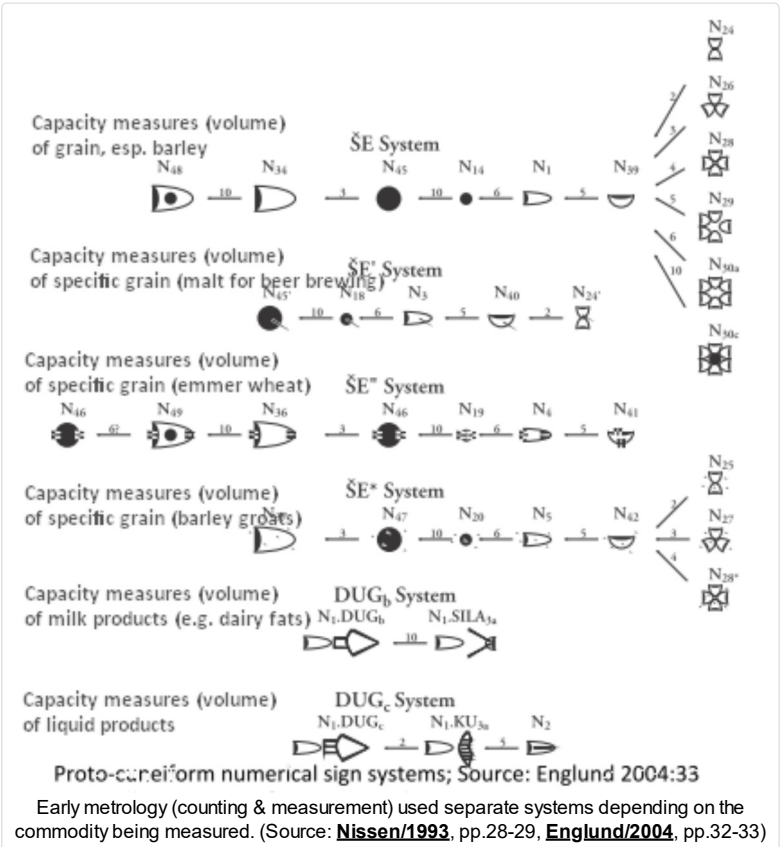


Figure 4a: Proto-cuneiform numerical sign systems.
Several systems of numerical signs served to qualify discrete objects (Fig. 4a), while others qualified measures of grains, (semi-)liquids and time (Fig. 4a and 4b).

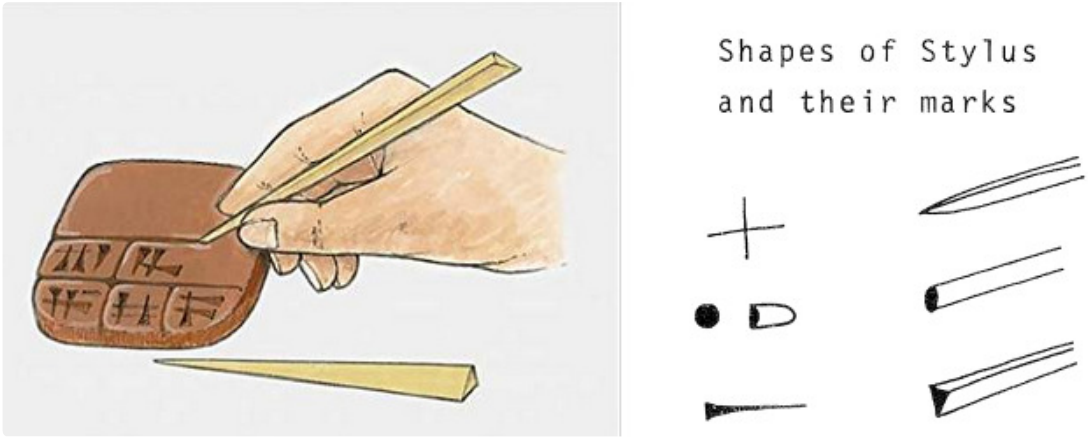
Early metrology (counting & measurement) used separate systems depending on the commodity being measured. (Source: **Nissen/1993**, pp.28-29, **Englund/2004**, pp.32-33)



Early metrology (counting & measurement) used separate systems depending on the commodity being measured. (Source: **Nissen/1993**, pp.28-29, **Englund/2004**, pp.32-33)

From 3,200 BCE onwards, there is increasing archaeological record of clay tablets [**Friberg/1984**] from which we have been able to piece together how the scribal-statal system evolved its written accounting practice

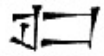
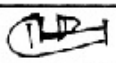
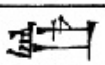
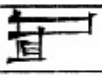
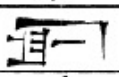
[**Hoyrup/1991**]. This proceeded from (1) an initial stage combining quantity and commodity in a single token/impression, to (2) using separate symbols for representing quantity (number) and commodity (pictographs), leading to (3) logographic Sumerian cuneiform (writing with the wedge-end of a reed) ca. 500 years later, and finally to (4) a syllabic representation of the Sumerian spoken language using cuneiform symbols (after ca.1000 years more) enabling Sumerian and Akkadian scribes to record concepts and literary ideas as well as numerical transactions. [**Nissen/1986**], [**Robson/2008**]



Style of writing evolved from pictographic to cuneiform as the stylus changed. (Source: **Nissen/1986**)

The evolution of writing over 1,100 years, from proto-cuneiform in Late Uruk period (3,100 BCE) to syllabic cuneiform during the Ur III period (2000 BCE)

The archaic texts from Uruk 321
Hans Nissen; 1986

Late Uruk Period ca. 3100	Jamdet Nasr Period ca. 3000	Early Dyn. III Period ca. 2400	Ur III Period ca. 2000	Meaning
				SAG 'Head'
				NINDA 'Bread'
				KU 'to eat'
				AB 'Cow'
				APIN 'Plow'
				KI 'Place'
				'10' resp '6'
				'1'

The evolution of writing from pictographs/logographs to cuneiform, as the type of stylus changed. (Source: **Nissen/1986**)

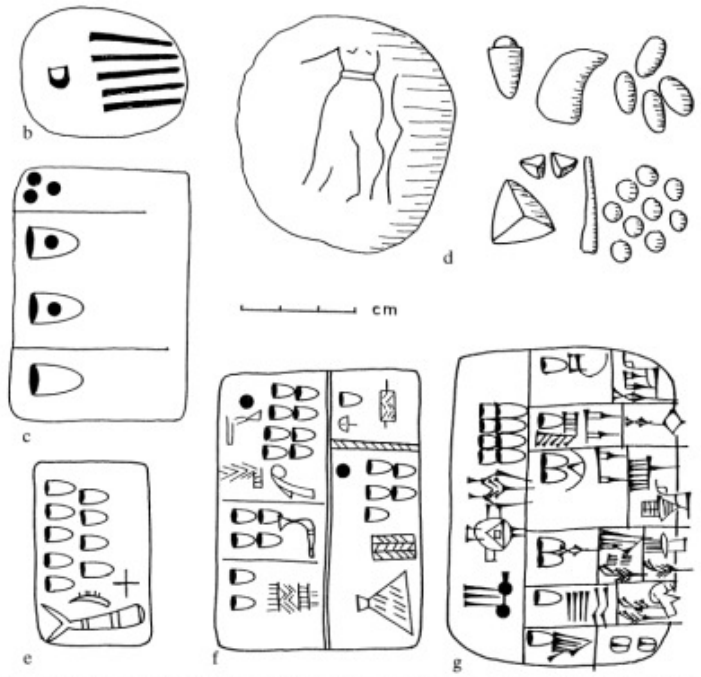
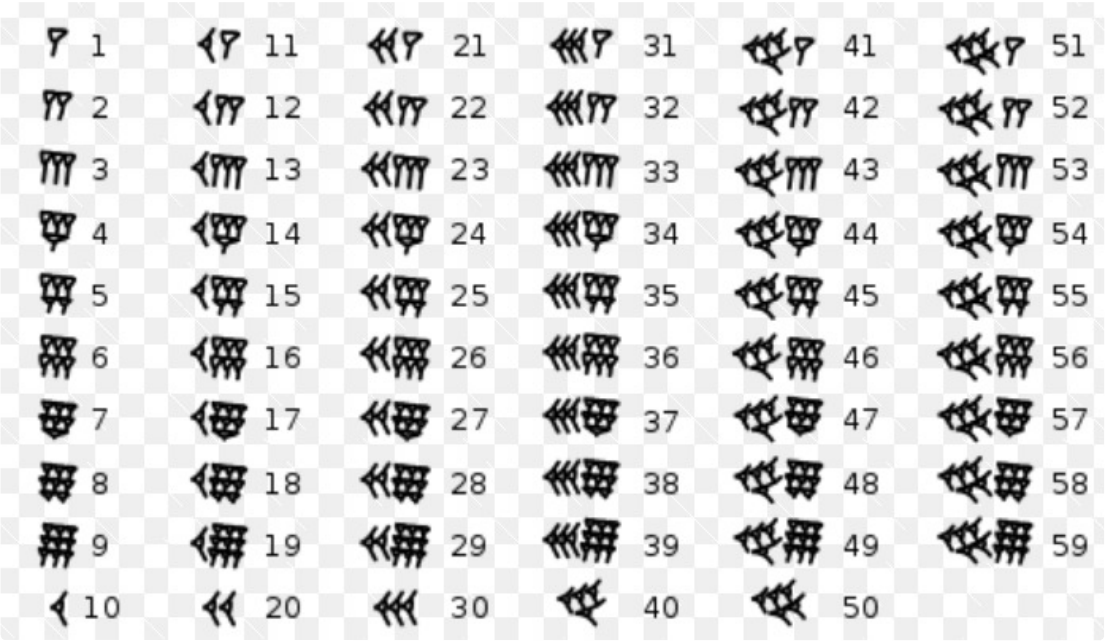
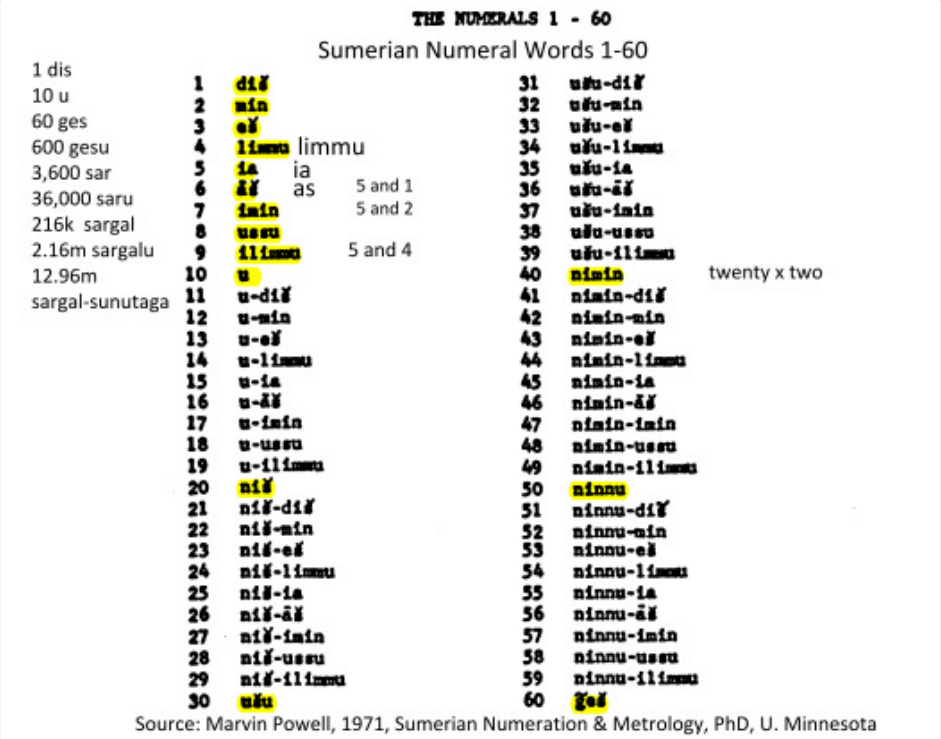


Figure 4 Early controlling devices: (a) Cylinder seal and sealing; (b) simple tablet containing numerical signs only; (c) complex numerical tablet divided into several compartments; (d) clay 'envelope' and clay tokens; (e) simple economic tablet of stage IV; (f) complex economic tablet of stage IV; (g) complex economic tablet of stage III 2.

The complexity of administrative tablets and notation evolved from quantity/commodity combined (commodity implicit) to quantity and commodity indicated with separate signs. (Source: **Nissen/1986**)



Cuneiform numerals 1-59, later than 2900 BCE (Source: **Powell/1971**)



Sumerian numeral words for the counting system (sexegesimal). Note the alternating x10 and x6 factors, all the way past 12 million. Source: **Powell/1971**, p.47

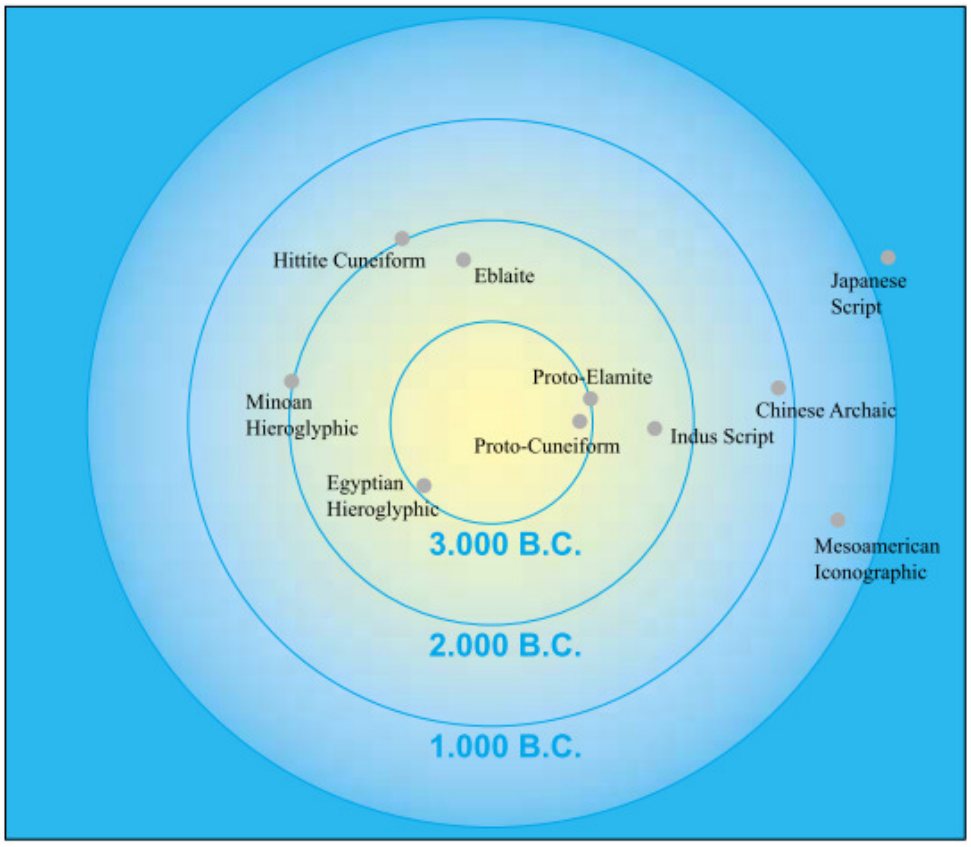


Figure 1. Historio-Geographic Map of Earliest Attestations of Writing

Earliest attestations of writing, beginning with proto-cuneiform c.3200 BCE (Source: **Damerow/1999**)

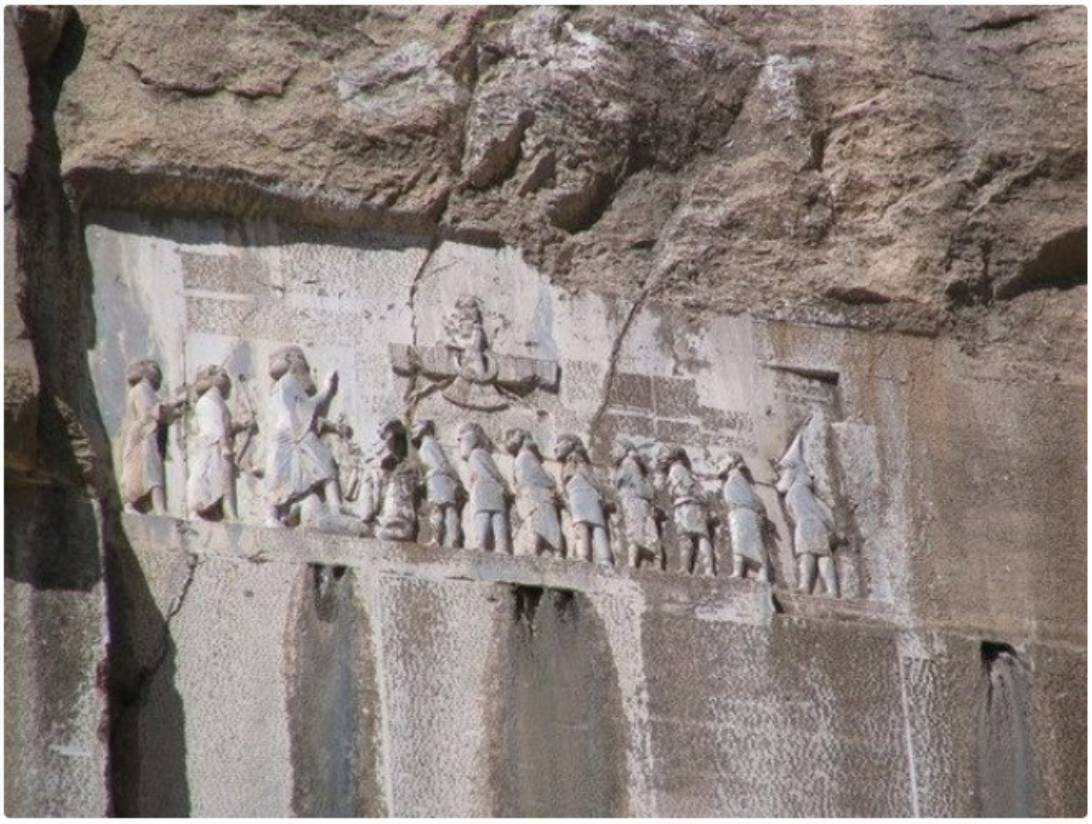
The deciphering of cuneiform languages: Sumerian, Akkadian, Babylonian, and Old Persian

How do we know what these signs mean? Our understanding of cuneiform is relatively recent, with pioneering deciphering work beginning from 1838 (CE) onwards [**Friberg/1984**]

Behistun: from Herodotus to Old Persian to Babylonian

The trilingual **Behistun Inscription** is to cuneiform writing what the rosetta stone (discovered 1799) is for the understanding of Egyptian hieroglyphics. The Inscription was engraved into the face of sheer cliffs near the ancient crossroads of Behistun (Kermanshah province of Iran) by the Achaemenid (Persian) king Darius the Great (Darius I) in 550 BCE. Its message proclaiming his conquest of all the lands and his right to rule, was intended for the entire Near East, and so was written in the three major cuneiform languages of his day: Old Persian (his own language), Elamite (the language of Susa), and Babylonian (the semetic language understood across Assyria and Mesopotamia). All three languages had died even by 400 BCE and over the millenia fanciful suggestions were put forth as to what the inscription signified.

In 1802, Grotefend had deciphered ten of the 37 symbols of Old Persion. Sir Henry Rawlinson started in 1835 using Grotefend's efforts. He found the first part of the Inscription contained the same list of Persian kings as given in Herodotus (400 BCE) but in their Persian forms. By 1838 Rawlinson had succeeded, in part due to the fact that Old Persian used an efficient syllabic representation of 37 characters. In 1844 and 1847 he studied the Babylonian section. Edwin Norris, a colleague, completed the study of the Elamite section by 1855. By 1855 [**Rawlinson/1855**] and Norris with a few others (Hincks 1854) had deciphered all three cuneiform sections: Old Persian (37 characters), Elamite (131 characters) and Babylonian (500 characters, more than 10x the number for syllabic Old Persian).



Trilingual Behistun Inscription carved into sheer cliffs in the Kermanshah province of Iran, engraved c.550 BCE by the Achaemenid king Darius the Great.

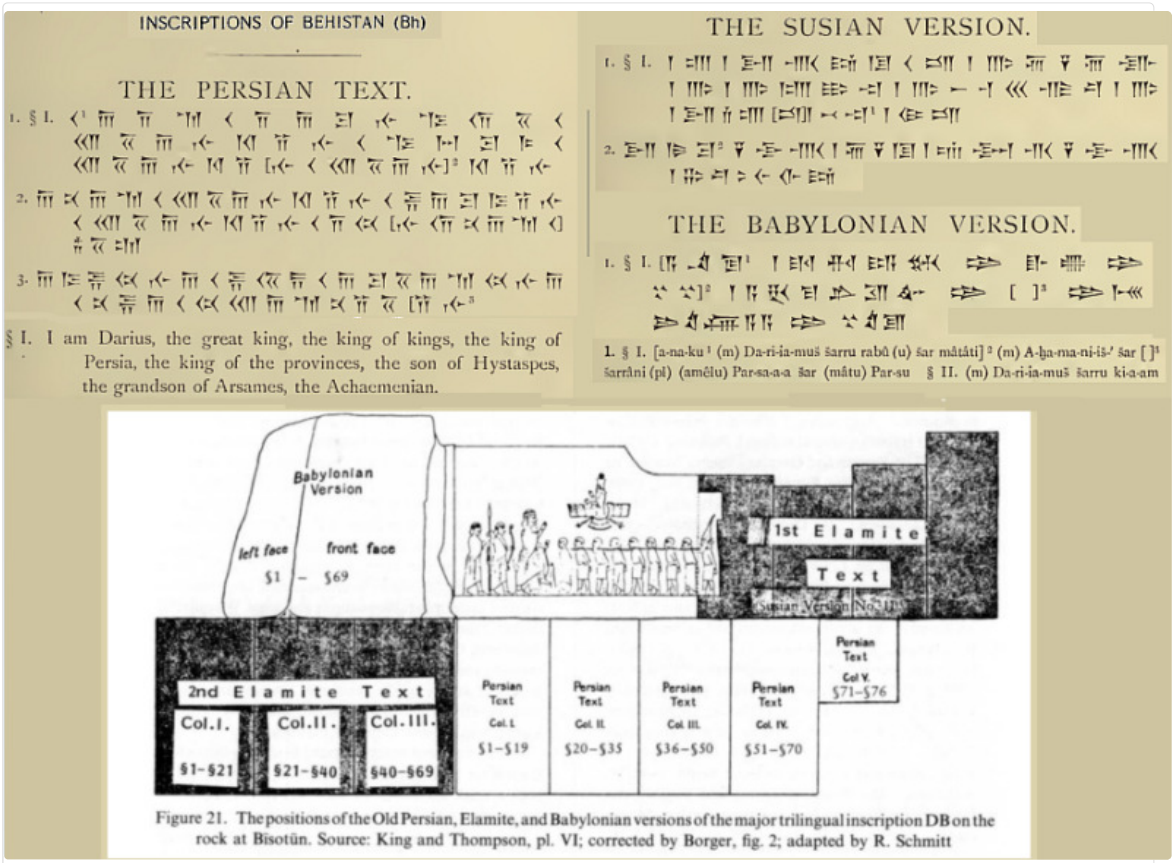


Figure 21. The positions of the Old Persian, Elamite, and Babylonian versions of the major trilingual inscription DB on the rock at Bīsotūn. Source: King and Thompson, pl. VI; corrected by Borger, fig. 2; adapted by R. Schmitt

The trilingual inscription at Behistun commissioned by Darius the Great in 550 BCE with messages written in the three cuneiform languages of the time: Old Persian, Elamite, and Babylonian (Source: **LW King & RC Thompson, 1907**

From Old Persian to Babylonian to Akkadian to Sumerian

Behistun provided the key to Babylonian through **Old Persian**, which is accessible through Middle and Modern Persian (Farsi). Babylonian (and Assyrian) are derivative dialects of an older Semetic language **Akkadian**. Their decipherment was completed by Hinks, Rawlinson, and Oppert in the mid 1800s, and from 1956 through 2011, the 26-volume Akkadian dictionary was compiled by University of Chicago (freely available online). To get to **Sumerian** we needed to rely on Sumero-Akkadian bilingual texts, and fortunately there are many, primarily the sign lists written by the early scribes that lived during the time of the transition from Sumerian city states to the Akkadian Empire, after Sargon unified the Sumerian city states under his rule.

From Sumerian cuneiform to proto-cuneiform (pictographs)

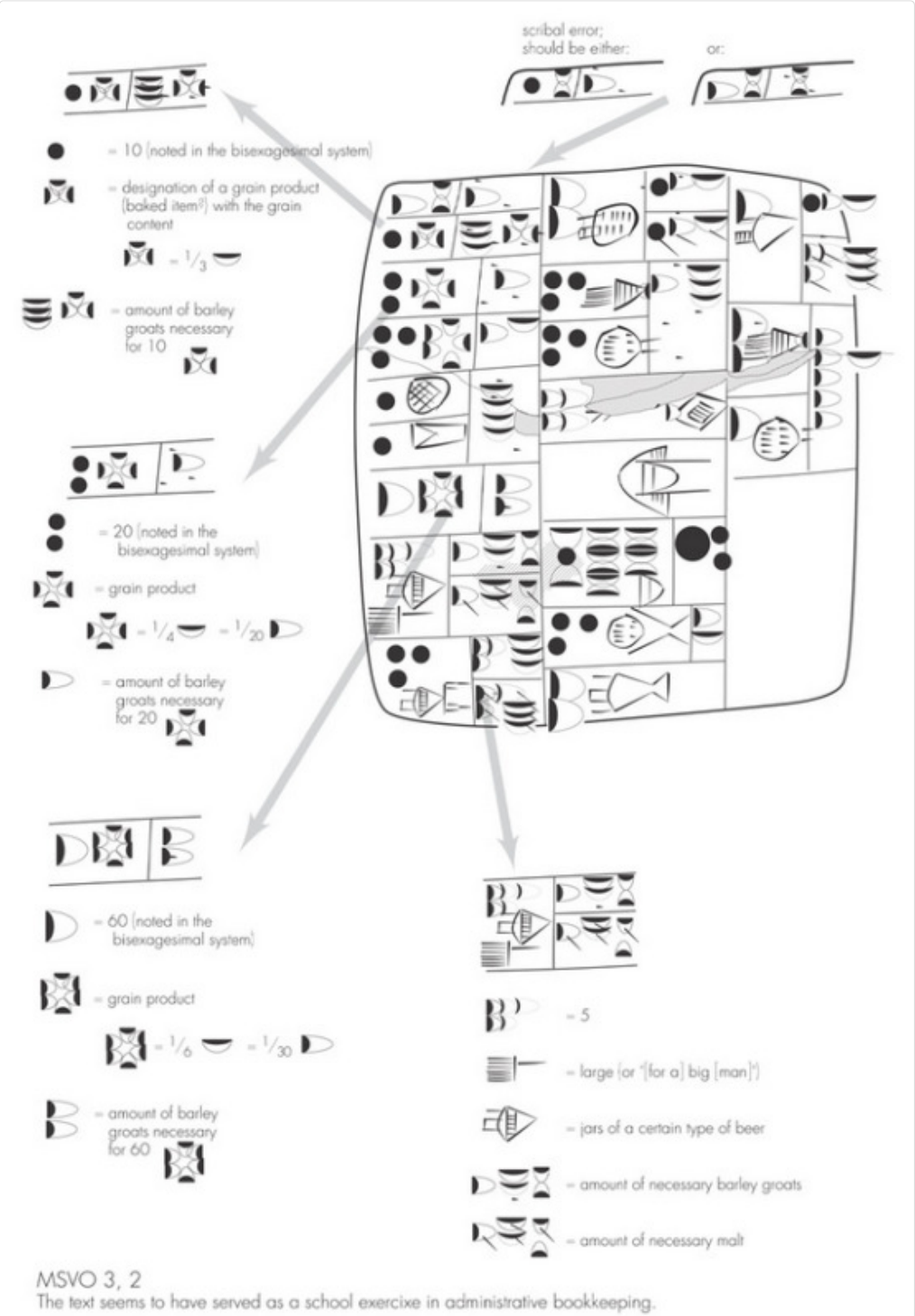
Deciphering an unknown but syllabic written language is hard. Deciphering the meaning of pictographs is harder still. To get to the meaning of the proto-literate writings it has taken the efforts of the Berlin group, a cross-disciplinary group of researchers who have used computer aided digitization of dozens of fragments to complete the work begun in 1936 by Adam Falkenstein who first published the Archaic Texts of Uruk (ATU).

Nissen 1986 (p.317) explains: even in 1936 it was recognized that a few texts were lists. Later more lists were discovered. And it was noticed that these proto-cuneiform lists corresponded word for word, position for position with the same lists almost 600 years later which were written in Sumerian cuneiform which by this time we did understand.



Figure 3. Bevelled-rim bowl (left) used for the disbursement of rations represented by the sign NINDA (middle, left column) which could be used for designating a ration of a certain size or in a semantically defined sign combination for designating an institution (middle, left column). In combination with a man's head it formed the sign combination GU_7 (right) which later stood for the word „eating." In proto-cuneiform writing, however, this sign combination was exclusively used to represent a certain type of administrative activity related to the disbursement of rations.

Deciphering pictographs. Here: NINDA (bread, ration) and GU7 (to eat, distribute ration) (Source: **Damerow/1999**)



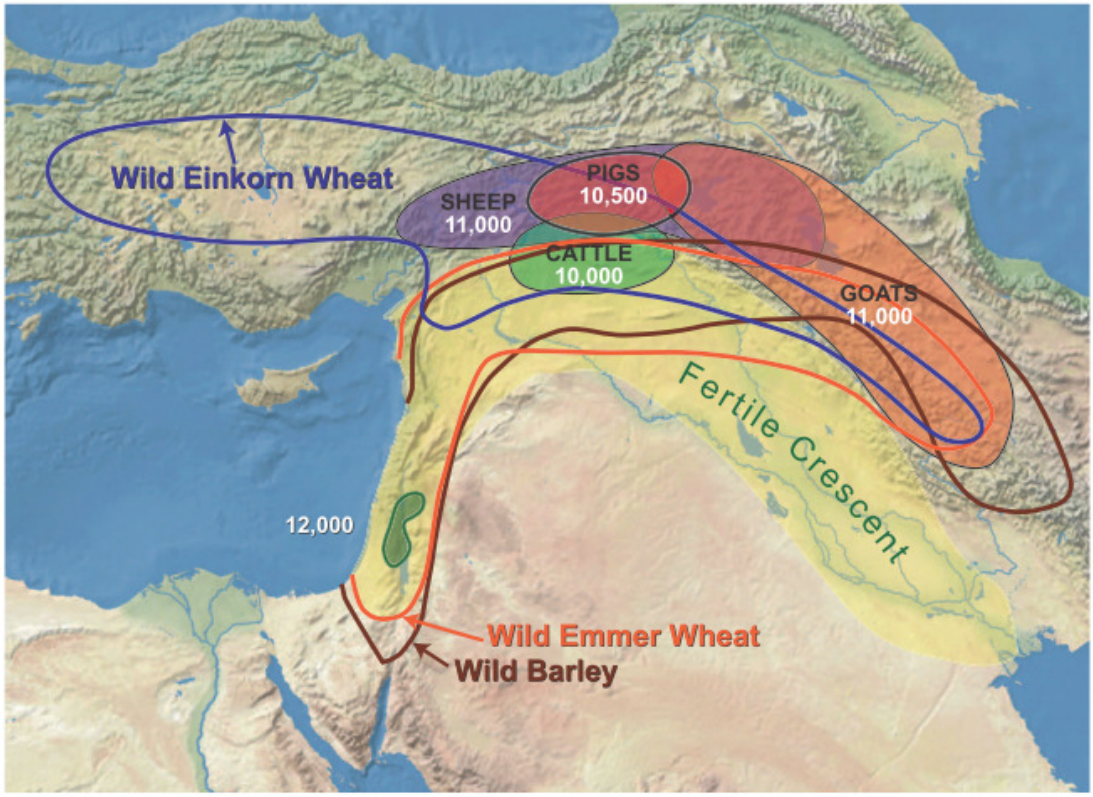
Interpreting pictographs in early accounting texts. (Source: **Englund/1998**)

But our interest lies earlier than 3,200 BCE, so we will leave dawn of written mathematics for now (but see the Reading List to continue) and move back in time.

2. Mathematical practice in the transition between Neolithic to Chalcolithic (Ubaid period): evidence from 6,000 BCE

Anthropological context

The formation of settled society occurred from 12,000 BCE to 10,000 years, with evidence for the deliberate cultivation of crops occurring c.9,000 BCE. This coincides with the time after the last ice-age receded from the Near East (c.12,000 BCE).



The neolithic revolution that followed the retreat of the 4th glacial ice age ca 12,000 years ago.

The practice of mathematics, counting and keeping time, shape and symmetry in craft, practical matters of building and measuring, appear to have begun at the same time as humanity progressed beyond the first stages of settled life into the more sophisticated cultural practices associated with agricultural surpluses and craft specialization.

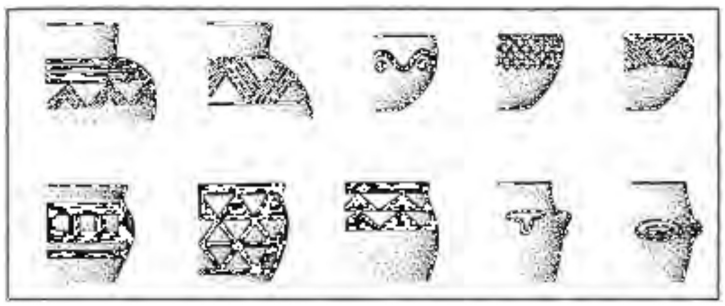
The earlier development of mathematical practice appears to have occurred during the time of transition in the Near East away from small Neolithic settlements that combined cultivation of crops and management of domestic livestock (primarily sheep and goats) within an egalitarian social structure, to the Neolithic-Chalcolithic Ubaid period, with larger size settlements, semi-permanent dwellings, increasing specialization in crafts, and evidence of hierarchical social status. [**Charvat/2002**] (See **Appendix 5** for more details on what life was like then.)

By the end of the Ubaid and at the start of the Uruk period, settlements were for the first time able to generate significant food surpluses through centralized management of labor resources directed to building large-scale irrigation systems to improve food output. The resulting economic and social changes, the rise of an established urban elite, and the shift to a redestributive, centrally controlled temple-economy drove the use of tokens as accounting devices, as we have seen above. [**Charvat/2002**], [**Niemi/2016**]

But even before this transition from settlements to city-states, there is archaeological evidence dating to 6,000 BCE (the Ubaid period) to support mathematical understanding and practice:

(1) painted pottery dating from 6,000-4,000 BCE show designs that use complex mathematical symmetries, and rotational frieze patterns, providing evidence for strong geometrical stylisation [**RobsonSelin/2000**].

Hassuna culture: painted and applique designs on pottery from the Yarimtepe I site (in Iran):



Painted pottery from Hassuna culture Yarimtepe I site dated c6500-5000 BCE. (Source: **Chavrat/2002** p.23)

Samarran culture: Pottery from Samarra from 6000 BCE-4000 BCE show confidence in geometrical form:



Samarran pottery, 6000 BCE-4000 BCE (Source: **Chavrat/2002**, p.35))

In the photo below of the Samarra Bowl (c.4000 BCE), we see:

> "Four stylised herons catch fish in their mouths while eight fish circle round them. An outer band of stepped lines moves outwards, countering the swirling effect of the animal figures." [**RobsonSelin/2000**]



Samarra Bowl (Pergamon Museum, Berlin), 4,000 BCE.
Painted pottery during the Ubaid period showing strong geometric stylisation (Source: **Wikipedia**, **RobsonSelin/2000**). Note Robson's image appears reversed from the Wikipedia .photo, making the flow clockwise instead of counter-clockwise

(2) analysis of **nine successive temple layouts** at **Eridu (first Sumerian city** mentioned in the King List) from Temples XVII c.5750 BCE through Temple VI, and comparison to other Ubaid period sites (6,500 BCE – 3800 BCE) show an architectural discipline in which prestige and communical buildings began to be laid out with increasing sophistication resulting in the use of modules with dimensions suggesting the use of a standardized length measure (Ubaid cubit of 0.72cm) as well as knowledge of side ratios of right triangles (3:4:5, and 5:12:13). [**Kubba/1990**], [**Forest/1991**] makes similar findings at Tell Hammam et-Turkman, **Soudipour/2007**.

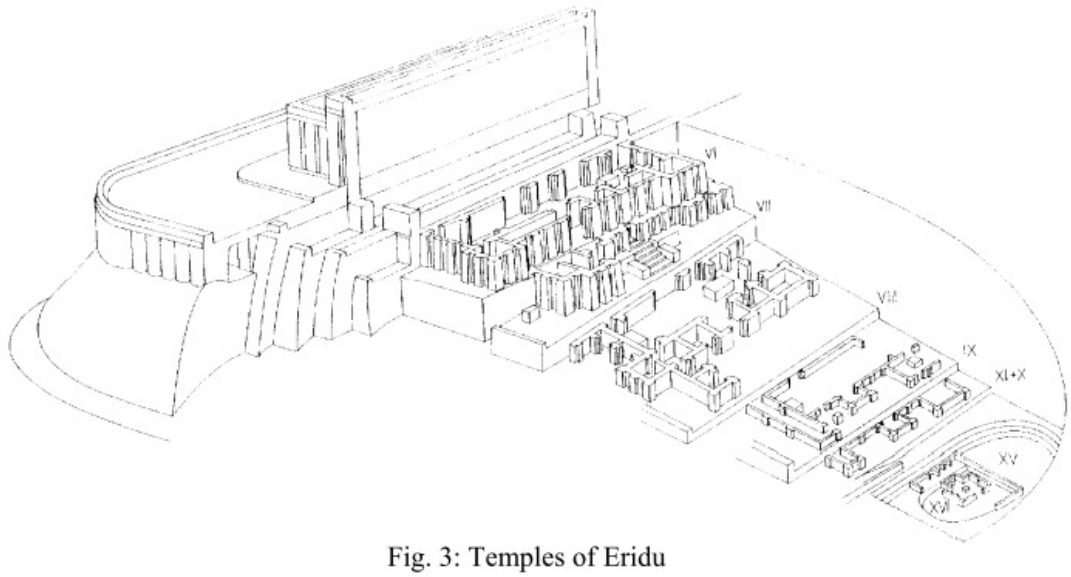
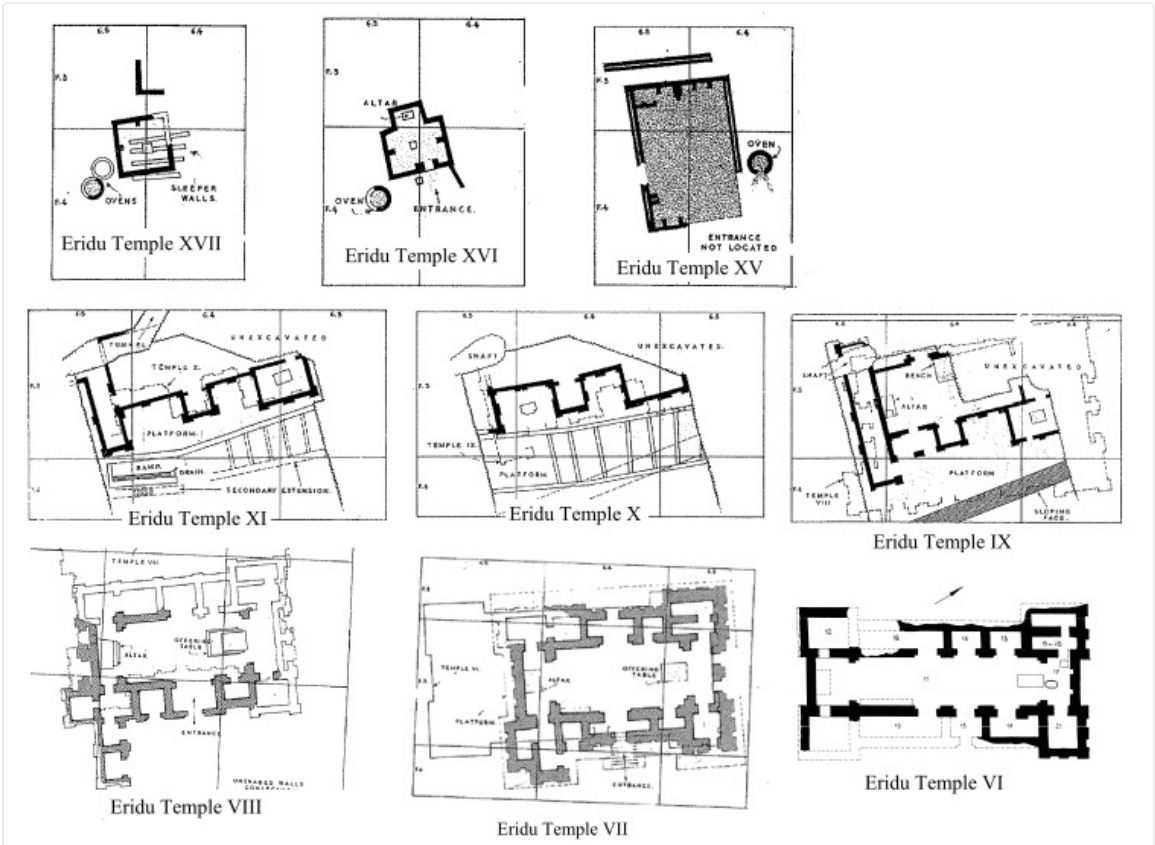


Fig. 3: Temples of Eridu

Temple layers at Eridu, built one on top of another over the course of 3000 years (5570 BCE through Ur III dynasty 2000 BCE) Source: **Soudipour/2007**, p.100



Nine floorplans from Eridu temples c.5570 BCE onward, showing increasing architectural sophistication until Temple VI with a distinctly mathematical layout (see Kubba 1990). Source: **Soudipour/2007**, p.101-102

From the earliest temple site (Temple XVII), there is a fixed orientation to all the buildings with corners at compass points N,E,S,W, creating a northwest-southeast axis.

> "The fact that buildings were built in such a way that their corners were made to face the central axis indicates an excellent knowledge of climatic matters. When the corners of the building face the north-south axis, the four sides of the building receive maximum sunlight, the northeastern and south eastern wall receive the sunlight until midday and the northwestern and the southwestern walls receive the sunlight after midday. Thus, all four sides of the building receive sunlight daily" (Youkana, 1997: 63)"

(3) While it is reasonable to expect that clay tokens were in use earlier than 3200 BCE for administrative purposes associated with temple management of Ubaid period economy controlling surpluses and labor, the contextual evidence around plain tokens finds across the Near East before 3,200 BCE is not conclusive, as there is insufficient context to conclude that they were used for counting and measurement, as occurred later in Uruk and Susa (see [**Niemi/2016**, pp.33-34, and Bennison/2018, pp.20-22).

The interaction between culture and mathematical development

In this and the previous period, what we have seen is that mathematical practice has arisen within a social context. It has been developed (invented?) and perfected within that social context for an application with a social purpose (accounting, recording of commercial transactions, state management of labor and food surpluses, design of prestige buildings, or the manufacture of status goods). Through its use, mathematics then affects and in many cases transforms the culture it arises within, and shifts it in new directions, which in turn affect the direction of further mathematical practice. See [**Hoyrup/1991**] and [**Robson/2008**] for examples of how the culture of the scribal schools varied from Uruk to Akkad to Ur to **Hammurabi**'s Babylon to the fall of Babylon, a period covering 1500 years.

3. Into the Paleolith: limitations of direct archaeological evidence, and a look at controversial claims dating mathematical practice to 70,000 BCE.

Can we find direct evidence of mathematical practice in the Paleolithic before humanity became settled?

There are two problems with older archaeological evidence: first is that many materials that may have been part of mathematical practice are bio-degradable and would not have survived (e.g. markings on sand with a stick, tallies on wood). Those that could survive (pebbles, bones) lack any cultural context to confirm mathematical usage (e.g. notches on bones could suggest tallying, arithmetic, or pre-historic calendar cf. [Marshack/1971] but could also have been made for a non-mathematical use, e.g. to improve the grip of the object) [Elkins/1996]

There are a handful of isolated finds without archaeological context and whose mathematical intention relies on interpretation of marks which remain tenuous and highly controversial (cf. Claim [Marschack/1971] and rebuttal [Elkins/1996]; Claim [Huylebrouke/1996], and rebuttal [Keller/2010]). Before 6,000 BCE there have been only a few older artifacts discovered that have been controversially associated with mathematical intention: (a) three notched bones from Uganda (Ishango bone), Swaziland (Lebombo bone), and Czech (Wolf bone) dated between 18,000 and 35,000 BCE, that suggest tally markings, and (b) an engraved red ochre lump from S.Africa dated to c.70,000 BCE suggesting geometric decoration.

But assigning mathematical intention to these is tenuous, because unlike the token/bulla findings or pottery and building layout findings above, if the markings on these artifacts are indeed symbols rather than practical scrapings (such as one might make to improve grip on a bone or cuts to release color powder from a lump of ochre).



Notched bone (Ishango) and engraved red ochre (Blombos cave). There is no context that indicates whether the markings have meaning.

This is the dubious circumstance that surrounds the famous notched bone artifacts dating between 35,000 BCE and 18,000 years ago. We do not have enough archaeological context on why or what they were carved for in order to interpret them. This has not stopped these interpretations making their way into the non-specialist literature covering ethno-mathematics and textbooks on mathematics history, where the interpretations have ranged from lunar calendars and fertility tallies, to multiplication tables and prime number lists. As an example, textbook historian David Burton follows Marshack and represents a current enthusiastic popularization when he writes of the Ishango bone: “It had been used for reckoning time “in sequences of numbers that agree with the number of days included in successive phases of the moon.” [Burton/1982] [Burton/1982], [Huyle/1996], [PletserHuyle/1999].

More critical recent scholarship has drawn important cautions (Elkins/1996) and Keller/2010):

> "The siren song of mathematical illusion is never far away when it comes to prehistoric artifacts. A notch may be nothing more than a mark, which seems like small fry if one is obsessed with arithmetic, yet it is the sign that is the most important invention we owe to our ancestors of the Upper Paleolithic. The common denominator of all the ethnographic artifacts of this kind is that they all show bijections, or an item by item symmetry between objects and signs. Faced with the raw artifacts of prehistory, it is impossible to know … whether the markings are decorative or not, and if they are not, whether we are dealing with an artificial memory system." (Keller/2010)

Elkins analysis [Elkins/1996] takes apart Marshack's microscopic readings of notched bone and highlights the repeated unjustified leaps in going from evidence to conclusion.

Thus, to go before settled times, we will need to consider indirect evidence.

One approach is to establish when humanity developed the symbolic capability to support numeracy.

4. Indirect evidence: the rise of symbolical capability in humans: 250,000 years ago

What evidence do we have of mathematical practice before the last ice age receded? Did humans of the Paleolithic need basic quantitative understanding before 12,000 BCE. For context, the domestication of the dog occurred c.18,000 BCE. Taking the long view, this phase between Paleolithic and Neolithic is the earliest 0.5% of human presence on Earth (which stretches back to 3.5 million years). (See **Appendix 5** for details of life to the end of the paleolithic.)

By the time of the fourth glacial advance 100,000 years ago (Upper Pleistocene), humans (Homo Sapiens) had emerged and domesticated the dog, dressed and sewed skins, were able to live beyond the frost line, had a culture of arts and crafts and a ceremonial society that buried the dead and showed solicitude to the aged and maimed. Archaeological evidence shows that intelligence, communication, and social living stretch back to 250,000 years ago (Middle Pleistocene), when humans had already evolved into what is essentially their modern form, Homo Erectus, and were using speech, tools, fire for warmth and cooking, were hunting large adult animals, and had diversified into all of the major races. Presumably, then, there would already have been utility in comparing, for example, the number of men in a hostile encampment with those in the home group, and in communicating this numerical information for group action. Similarly, a builder or toolmaker needing material for a particular purpose would have needed to specify dimensions, even if roughly. An elder needing to know how long a hunting party had been absent before setting off to investigate would have needed to mark time.

Until early in the current century, the prevailing opinion was that humankind developed symbolic capability between 50,000 year ago and 250,000, coincident with the emergence of anatomically modern man (Homo Sapiens). This was based on:
(1) the discovery of the earliest human art (cave paintings, jewelry/decorative power),
(2) anthropological evidence of ritual burial of the dead,
(3) anthropological evidence of cooperative hunting which presumes the ability to communicate intentionally and with precision,
(4) the practice of language, indicated by earliest presence of the human version of the FOXP2 gene which regulates learning and complex speech, combined with the assumption that (a) the ability to speak implies that speech and language occurred, and (b) that any language, no matter how primitive, must be symbolic and include at least a rudimentary number concept (e.g. one-two-many, or even one-many). Certainly this last assumption was empirically correct based on the evidence of all known primitive languages encountered before the 1970s) (cf. [Levi-Conant/1897], [Smith/Ginsburg/1937] and [Gullberg/1997])

These views have changed in the past 20 years following the discovery of the Piraha people of the Brazilian Amazon, whose language surprisingly has no numerical concepts at all [Piraha/2006]. The Piraha (both the people and their language) provide experimental evidence for that critical link (heretofore missing) that there can be a state in evolution of language and culture between symbolic capability and numerical capability. [C.Everett/2016] [DL.Everett/2018]

Language and the Number Concept

The presence of speech has been considered a strong indicator of numeracy, since of all the known languages and cultures that have been studied, except Piraha, have numeric concepts. To wit: “we know of no language in which the suggestion of number does not appear, and we must admit that the words which give expression to the number sense would be among the early words to be formed in any language.” [Conant/1896], [SG/1937], [Piraha/2006], [Piraha/2007], [Gordon/2004], [FEFG/2008], [EM/2012] The language and culture of the Piraha people is the first known counter-example. Their unusual language has no numbers (not even the “one”-“two”-“many” pattern found in other primitive languages) and this is changing our view of what language is and how it may have evolved.

Studies of the Piraha suggest that numerical capability appears to require three things: (1) the capability for symbolic thought (e.g. grasping the notion of bijection, which underpins discrete comparison); (2) a mechanism to keep the count (e.g. fingers, marks/notches, pebbles, or linguistic counting words), and, taken for granted before the Piraha, and, most importantly perhaps, (3) that the embedded culture assigns *value* to numeracy.

The Piraha, in particular, have no value for number in their culture, and so have not only not developed any mechanisms for counting, but actively resist the learning and retention of these mechanisms [Gordon/2004], [Frank.DLEverett/2008], [C.Everett/2016].

The third point places the development mathematical practice centrally within cultural context. We cannot of course overlook the importance of fundamental neurological ability for symbolical thought, nor the development of a mechanism for keeping the count. While one may indeed grasp the notion of bijection, without a mechanism to keep a precise tally one cannot actually count, only match. (How the tally itself is made is less important and can take many forms: visually by using fingers of the hands or creating marks or notches, physically by collecting pebbles or other tokens or calculi, or linguistically using by words and/or signs.

But without the third (valuation of counting), the case of the Piraha show that humans essentially fall back on what appears to be a biologically innate analogue number sense that is also present in animals, birds, and even some reptiles, but which decreases in precision as magnitudes get larger. (This is why animal counting degrades quickly beyond four or five). [Gordon/2004] [Everett/2012], [Dehaene/1997]

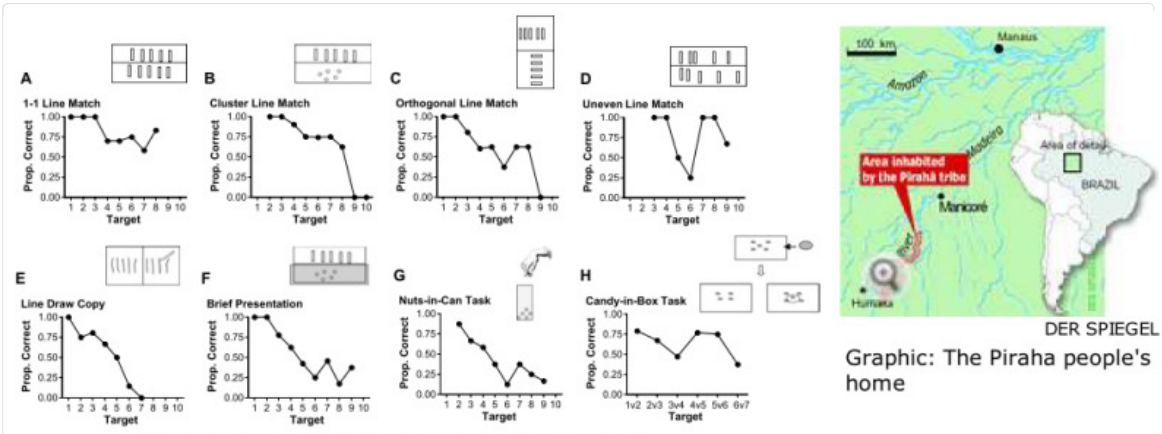


Fig. 1. Results of number tasks with Pirahã villagers ($n = 7$). Rectangles indicate AA batteries (5.0 cm by 1.4 cm), and circles indicate ground nuts. Center line indicates a stick between the author's example array (below the line) and the participant's attempt to "make it the same" (above the line). Tasks A through D required the participant to match the lower array presented by the author using a line of batteries; task E was similar, but involved the unfamiliar task of copying lines drawn on paper; task F was a matching task where the participant saw the numerical display for only about 1 s before it was hidden behind a screen; task G involved putting nuts into a can and withdrawing them one by one; (participants responded after each withdrawal as to whether the can still contained nuts or was empty); task H involved placing candy inside a box with a number of fish drawn on the lid (this was then hidden and brought out again with another box with one more or one less fish on the lid, and participants had to choose which box contained the candy).

Experiments on Piraha numeracy (Source: Gordon/2004); (right) Piraha homeland (Source: Der Spiegal/2006)

The evidence for speech

What evidence exists for speech? Genomic investigations into speech defects have identified the **FOXP2 gene** as a critical link to and enabling factor of speech control. Absence leads to non-viability, reduction leads to significant vocal disability. While the FOXP2 gene is expressed in birds, mice, primates, and humans, the human variation is different from all the others. The modern functioning version has been present in humans between 120,000-260,000 years ago, either the last common ancestor of Neanderthals and Homo sapiens, or specific to Homo sapiens. If we allow the assumption that the capability for complex vocalization means the realization of speech and language, and furthermore associate language with symbolism and the number concept, we arrive at a similar date of 260,000 years ago.

5. Geologic and Paleo-anthropological context before 250,000 years ago

Can we say what came before 250,000 years ago? Geologic timelines show the extinction event for land dinosaurs at 66 million years ago (mya), after which a slow cooling off period began, accompanied by the proliferation of mammals. From 34 mya to 23 mya the Earth transitioned from a tropical world to modern ecosystems. From 23 mya to 2.6 mya the cooling continued. Primates were already living in trees by 13 million years ago, and hominids had branched off between 7.5 to 5.6 mya. At 2.6 mya, the four ice ages began (Pleistocene period) with the last glacial retreat occurring around 12,000 years ago (12 kya) and the inter-glacial warming period (holocene) beginning 10 kya. [Coon/1996].

Download **Timeline (PDF) of Early Human Life, from 55mya to 5kya (Tom Conklin, 2009)**

By 2.6 mya, the fossil evidence shows sufficient abstract capability and communal sharing of knowledge had developed in early hominins to allow the rise of the Oldowan tool-making culture, which continued for the next 900,000 years when it was replaced by the Acheulean culture of late Homo Erectus (1.7 mya). Icon establishment (recognition and consistent interpretation) was based on association of meaning with form. Progress to the last semiotic stage, i.e. symbol creation (arbitrary signs with conventionally agreed interpretations providing meaning, and a grammatical structure for language) was likely not till 250,000 to 50,000 years ago, coincident with the bigger brains of Homo Erectus culture coincident with the Acheulean advances in tool-making.



Oldowan Tools



Acheulean tools:

Stone Tool Cultures of early Hominids: Oldowan (2.6mya) and Acheulean (1.6mya)
(Source: EverettDL/2016)

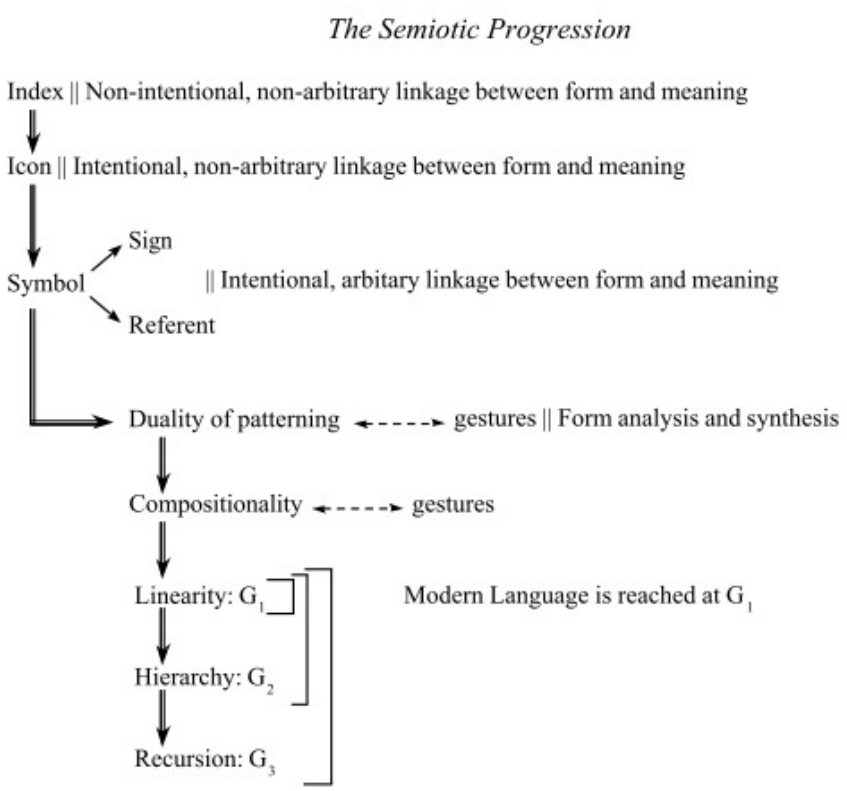
6. Semiotic model for linguistic and conceptual development: distinctly human achievement to 2.4m years ago.

The importance of the Piraha to theories about prehistoric language development and numeracy is that they provide field evidence to support the gradual notion of language development in humans rather than this being the result of a discontinuous watershed moment in history (whether genetically, biologically, or neurologically). In terms of models, this favours C.S. Pierce's semiotic model over Chomsky's linguistic characterization model. In Pierce's model [Everett/2017], humans (and also intelligent animals) start with the ability to interpret of physical "indexes" (stage 1). The next stage (for which currently we only have evidence of humans reaching) is the use and understanding of icons which are intentionally chosen to represent physical phenomena and hence are relatively easy to interpret (e.g. picture of a cow, picture of fire, etc.). The final stage is the use of symbols, i.e. signs that are arbitrary and require establishment by cultural convention in order to interpret.

Overlaying the semiotic model with archaeological data [cf. Turkana/2005], Everett argues that symbolic capability in humankind may indeed date back almost 10x further than the earliest Chomskian estimate, to 2.4 million years ago .

A closer look at semiotic progression: an alternative perspective to the acquisition of language

In the last 20 years, an alternative, fieldwork approach to linguistics (as opposed to theoretical work of Chomsky) has challenged the interpretation of language acquisition in early humans. Language as a means of communication is argued to have evolved from the earliest hominins 2.6 million years ago through a semiotic progression from indexes to icons to signs and symbols (C.S. Pierce, Daniel Everett).



Semiotic Progression model of language acquisition, following C.S. Pierce (Source: Everett DL, 2016, p.18)

Index conditioning (Stage 1) is the ability of creatures with a nervous system to perceive an "index" (physical stimulus) and produce an appropriate response (e.g. recoil from a hiss, be wary of yellow and black insects, recognize footprints or smells). We have experimental confirmation that a variety of animals, birds, and reptiles possess the cognitive precursors of mathematics (number sense and the perception of shape, time, and change) as part of their biological index response capabilities. More advanced creatures can remember, learn, and adapt (i.e. animal training is advanced, but is still index conditioning). Icon communication (Stage 2) involves the intentional use of signs ("icons") chosen because of their close association with the intended physical meaning (e.g. smoke for fire, a figurine for motherhood, a stick drawing for a person, or a footprint or smell for the creature that caused it). Symbolic communication (Stage 3) involves the intentional selection of arbitrary signs whose meaning is established by cultural convention (e.g. male/female signs, traffic light colors, arbitrary gestures, tallies, arithmetic signs, numerals, etc.).

The archaeological overlay for semiotic view of human conceptual development

From the archaeological evidence it is clear that the first crude-tooled ancestors of man (Australopithecus) of 700,000 years ago and the earliest hominins 2.6 million ago, shared a language with which they could communicate with each other, but it is unlikely that it was mathematical: notches, marks, and counts all would need to rely on one-to-one bijection concept and arbitrary symbolism associating the notch with whatever was being counted.

7. Neurological basis for the "number sense" in humans and animals: back to 150 million years ago

We have seen above that the Piraha show that humans essentially fall back on what appears to be a biologically innate analogue number sense that is also present in animals, birds, and even some reptiles, but which decreases in precision as magnitudes get larger. (This is why animal counting degrades quickly beyond four or five). [Everett/2012], [Dehaene/1997]

If we consider as a precursor for mathematics the analogue number sense that we see in the Piraha, in babies,

and in humans when they are not allowed to use language or count, then we can trace this back another 100x earlier using the fossil record and paleobiology of modern animals that are known to have this number sense. Primates are dated from 13 million years ago, birds from 150 million years ago, and reptiles from at least 250 million years ago, and these were the last common ancestor of both birds and animals.

Number Sense in Animals

If we broaden our inquiry beyond the evidence for mathematical *understanding and use*, and into the possession of the cognitive precursors for mathematics, we find that both number sense (but not measurement or counting per se) and the perception of shape and change (though perhaps not their description or communication) are not unique to the human species. Investigations have found evidence of number sense in animals (birds, dogs, monkeys, dolphins). Perception of the passage of time, the ability to distinguish one from many (in particular, quantities other than two), and the ability to distinguish shapes from each other, have all been documented in various animals. [Koehler/1950] Intelligent primates go back to 13 million years ago, birds to 150 million years ago, taking us back to the last common ancestor of birds and animals, which would have been a smart lizard.

Dating the capability for mathematical cognition then becomes a question of the timeline of intelligent, perceptive life itself. The intelligent tree-dwelling primates of 13 million years ago certainly had the mental capacity for the cognition of the precursors of mathematics, but it is unclear whether they, like all other perceptive creatures that can respond to stimuli), are able to progress from the lowest rung of Pierce's 3-step evolution (indexes) to the next rung (intentional use of icons). While experiments have shown that animals can proceed from icon to correct decision (this is learning and cataloguing new indexes e.g. Pavlov's dog, trained monkey, crow), the challenge not yet demonstrated (as far as I am aware) is of an animal taking the stimulus, and picking the right descriptive icon. For the third semiotic stage no evidence that I am aware of documents a non-human creature intentionally adopting a completely arbitrary symbol or sign, and whose interpreted meanings need to be established as part of a cultural convention.

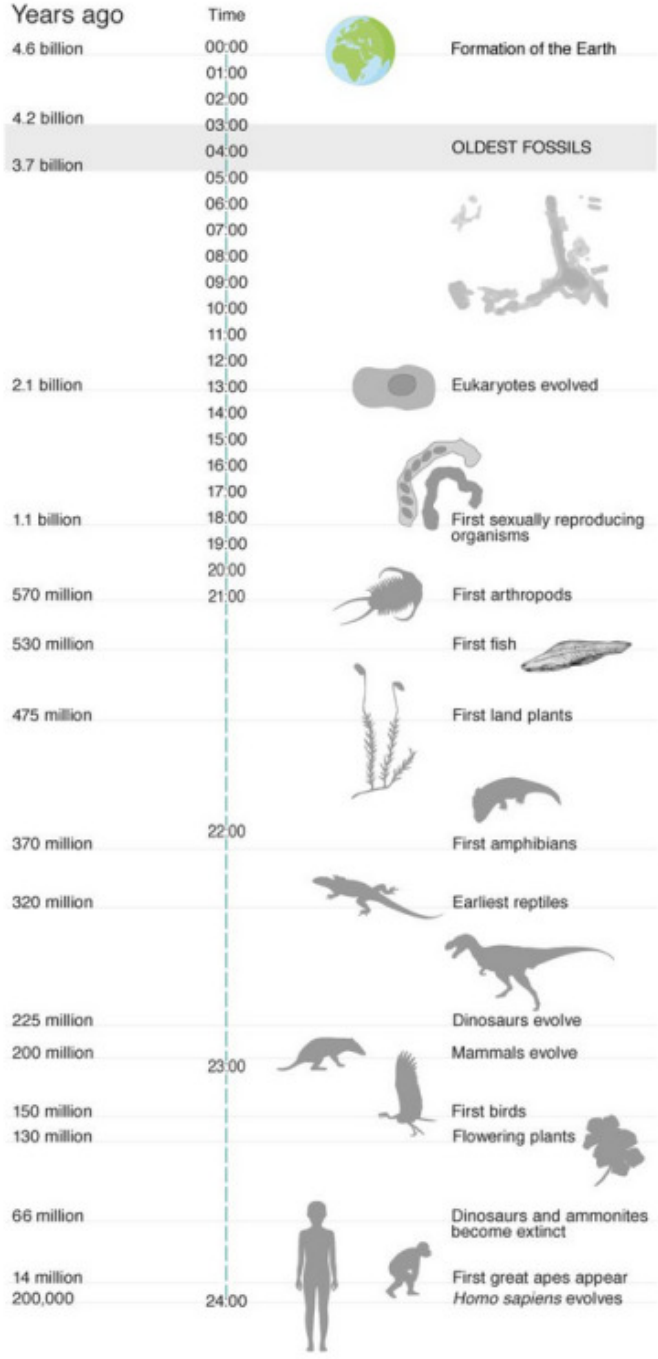
Neurological evidence

The semiotic perspective highlights that perception and response to indexes (unintentional physical associations) is common to all living things that can think (sense their environment and choose a response). And life on earth goes back at least a 3 billion years.

Is there an upper bound to cognitive perception of precursors of mathematics (number, shape, change, time)?

Neurological studies have established parts of the brain that mediate higher order functions such as sensory perception, spatial reasoning, learning and memory, decision making, motor control, and conceptual thinking. Studies show that these capabilities exist in birds (parrots) and even a few reptiles (monitor lizards), challenging the model that the mammalian neocortex (220 mya) is the seat of complex cognitive functions. In birds, it is the DVR (dorsal vernicular ridge) that provides neocortical-like functioning. Both the neocortex and the DVR have been found to develop out of the same region in the embryonic brain.

It is likely that the complex cognitive functions providing the origin of mathematical perception arose from a common ancestor of mammals and birds, i.e. a reptile existing some 150 million years ago, which would explain the presence of number sense in monitor lizards.



Timeline of Life on Earth. First birds at 150mya. Reptiles as the last common ancestor of birds and animals. (Source: Wikipedia)

8. Conclusions

The answer to the simple headlined question is far from simple.

- By 3,200 BCE (5200 years ago) there is indubitable evidence for mathematical practice in clay tokens and bullae (“envelopes”) in Mesopotamian city states within a centralized temple economy and a scribal-statal context. This is the earliest known system of metrology (counting and measuring), of writing, and of book-keeping (accounting)
- Around 6,000 BCE (8000 years ago) there is evidence of elaborate pottery with mathematical designs, disciplined building layouts showing the use of a standardized length measure and an understanding of principles of geometry including the application of right triangles.

- Around 250,000 years ago, we see in the archaeological record, changes in human species as anatomically modern humans emerge, and we have evidence of complex symbolic behaviour, which in principle could support numeracy.
- Around 2.4m years ago, we see in the fossil record the first appearance of technologically complex stone knapping tool-making by early human species, that support the progression of human-kind from index/response (shared with animals) to iconic thinking.
- 150m years ago neurological and biological evolution in reptiles had progressed far enough to allow index/response mechanisms that would qualify as "number sense" (though obviously not mathematical practice in any iconographic or symbolical sense).

How far back do I think mathematical practice began? Before answering, I will point to Newton's criterion for separating science from speculation: **"Hypothesis non fingo"** ("[I am certain if] I need feign no hypotheses.", 1713 paper) [Walsh/2010] But to answer: to satisfy a rigorous requirement for evidence, I believe we can only point to 6,000 BCE (8000 years ago). But considering the indirect evidence we have today, I believe the answer (though with no direct evidence currently known) is likely to lie somewhere between 2.4 million and 250,000 years ago. This may of course change as we continue to uncover windows into our prehistoric past. The story of prehistoric mathematics is undoubtedly not yet complete!

*Author's Note: An early version on the prehistoric origins of mathematics was written ten years ago (**Dec 29, 2009**). In the meantime, advances in linguistics, genomics, interpretive theory, and Mesopotamian mathematics itself, have made the story more complicated, more nuanced, and much more interesting. This second revision attempts to bring the question up-to-date and extends the **Reading List** (Nov 5th, 2019)*

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Appendices

Appendix 1. The Essential Nature of Arithmetic

The following are extracted verbatim from the outstanding essay of A.D. Aleksandrov, A General View of Mathematics, specifically section 5, pp.17-19, on the Essential Nature of Arithmetic in [**Aleksandrov/Arithmetic1956**].

1. How did the abstract concepts of arithmetic arise and what do they reflect in the actual world?
"The concepts [of arithmetic] arose by way of abstraction as a result of the analysis and generalization of an immense amount of practical experience. They arose gradually; first came numbers connected with concrete objects, then abstract numbers, and finally the concept of number, in general, of any possible number. Each of these concepts was made possible by a combination of practical experience and preceding abstract concepts. This, by the way, is one of the fundamental laws of formation of mathematical concepts: They are brought into being by a series of successive abstractions and generalizations, each resting on a combination of experience with preceding abstract concepts. The history of the concepts of arithmetic shows how mistaken is the idealistic view that they arose from "pure thought," from "innate intuition", from "contemplation of a priori forms," or the like." [Aleksandrov/Arithmetic1956]

2. Why are the conclusions of arithmetic so convincing and unalterable?
"We see that the conclusions of arithmetic have been worked out slowly and gradually; they reflect experience accumulated in the course of unimaginably many generations and have in this way fixed themselves firmly in the

mind of humankind. They have also fixed themselves in language; in the names for the numbers, in their symbols, in the constant repetition of the same operations with numbers, in their constant application to daily life. It is in this way that they have gained clarity and certainty. … What is essential here is not only the fact that they can be repeated at will but their soundness and perspicuity, which they possess in common with the relations among things in the actual world. This is the reason why the results of arithmetic are so convincing: its conclusions flow logically from its basic concepts, and both of them, the methods of logic and the concepts of arithmetic, were worked out and firmly fixed in our consciousness by [five] thousand years of practical experience on the basis of objective uniformities in the world around us." [Aleksandrov/Arithmetic1956]

3. Why has the abstract concept of number and arithmetic taken so long to arise?

"Every abstract concept, in particular the concept of number, is limited in its significance as a result of its very abstractness. In the first place, when applied to any concrete object it reflects only one aspect of the object and therefore gives only an incomplete picture of it. … It is impossible to apply arithmetic to concrete problems without first convincing ourselves that their application makes sense in the particular case. If we speak of addition, for example, and merely unite the objects in thought, then naturally no progress has been made with the objects themselves. But if we apply addition to the actual uniting of the objects, if we in fact put the objects together, for example by throwing them into a pile, in this case there there takes place not merely abstract addition but also an actual process, and in general it may be impossible to carry it out. For example, the object may break, wild animals if placed together may tear one another apart, materials put together may enter into a chemical reaction and so the sum e.g. of a liter of water and a liter of alcohol will not yield two liters of mixture but 1.9 as a result of partial solution of the liquids, and so on. To put it briefly, truth is concrete, and it is particular important to remember this fact with respect to mathematics exactly because of its abstractness." [Aleksandrov/Arithmetic1956]

4. What forces led to the development of mathematics?

"For arithmetic, the answer is clear from history. The forces that led to the development of arithmetic were the practical needs of social life. People learned to count and to work out the concept of number. Practical life, by posing more difficult problems, necessitated symbols for numbers. These practical needs and the abstract thought arising from them exercise on each other a constant interaction. The abstract concepts provide in themselves a valuable tool for practical life and are constantly improved by their very application. Abstraction from all nonessentials uncovers the kernel of the matter and guarantees success in those cases where a decisive role is played by the properties and relations picked out and preserved by the abstraction. In the case of arithmetic, this is the quantitative relations. … This is just a particular case of a phenomenon known to everyone, namely the interaction of experience and abstract thought, of practice and theory." [Aleksandrov/Arithmetic1956]

Appendix 2: The Origin of Writing

1. When was writing invented?

Proto-writing first appeared in the middle to end of 4th millennium BCE, c.3200 in southern Mesopotamia (Uruk and Susa). This was first purely numerical recording of quantities that had previously been recorded using tokens with the commodity understood from context; then the recording of the commodity separate from the quantity using pictographs; and then by end of 4th millennium, c.3200 proto-cuneiform in Uruk where both quantity and commodity were recorded in cuneiform encoded pictographs, followed by proto-Elamite in Susa c.3000. This led to then the standardization of cuneiform pictographs, followed by the next innovation c. 2500 in Fara where we see the early attempts at encoding phonetics to writing to reduce the number of individual signs needed and the burden on agreeing their meeting through cultural convention, as well as the application to the new conqueror language of Akkadian. See [**Damerow/1999w**].

Appendix 3: Summary: Mesopotamian Cultural History

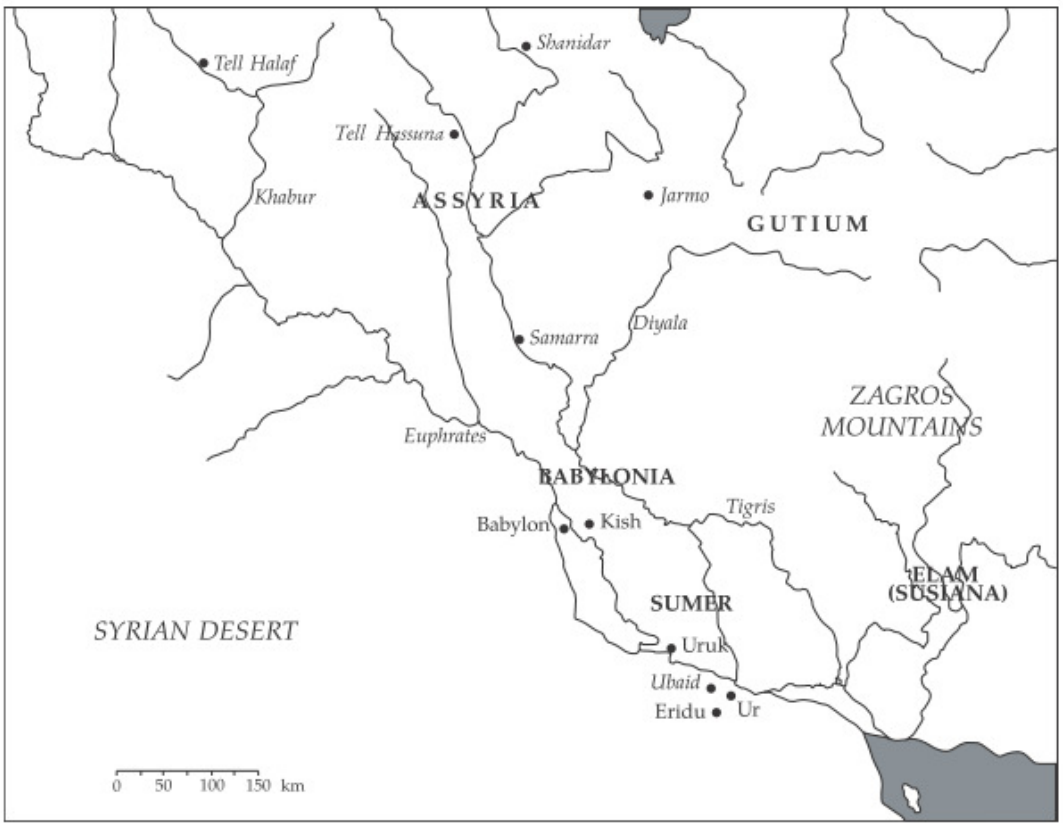


Fig. 1. The cities of ancient Mesopotamia mentioned in the text. Modern names in italics (drawing by Barbara Kasprzak).

Near East, from the Earliest Sites (Shanidar, Jarmo) to the Age of Empires (Assyria, Babylon, Elam, Guti)

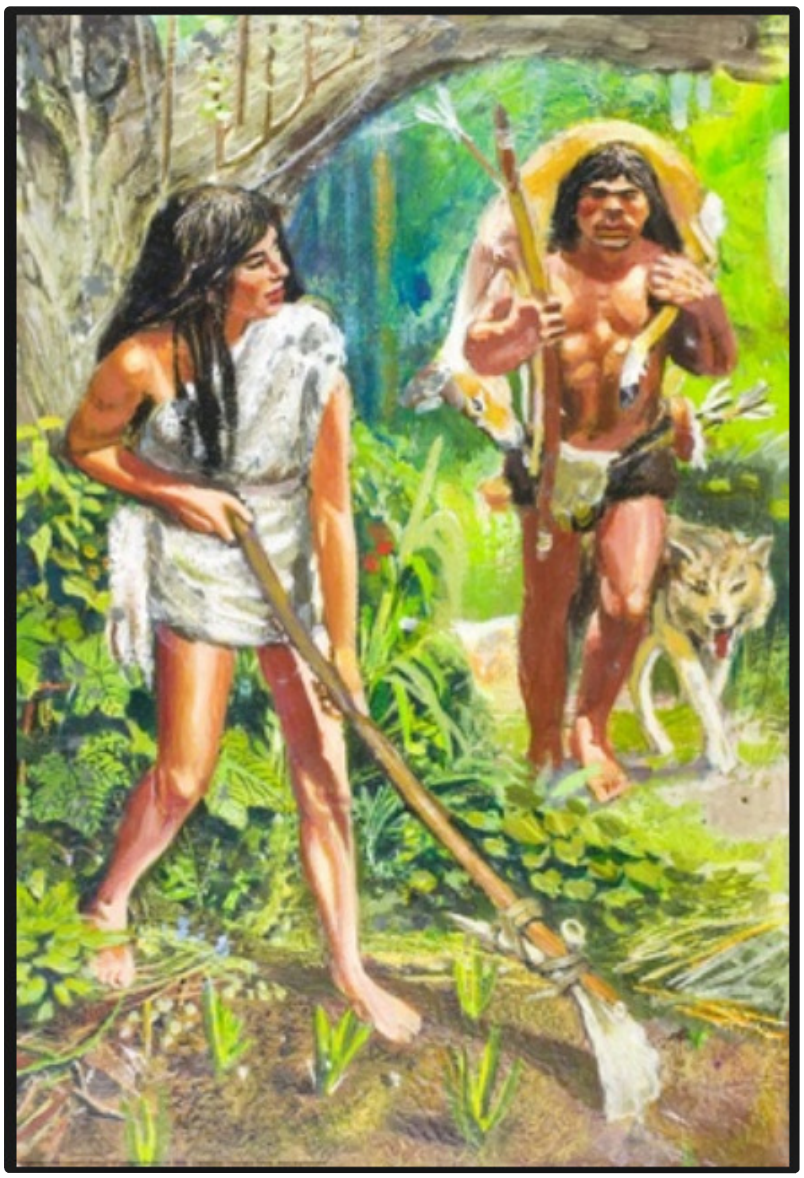
- **Paleolithic** – from the first stone tool wielding hominids (3.3 mya) through the four ice ages to hunter-gatherer subsistence mode (Neanderthals and Homo Sapiens) e.g. in **Shanidar Cave (65,000 BCE)**, a Middle Paleolithic (Mousterian) site in the Zagros mountains of Iraq/Iran/Turkey border. The last stage is **Upper Paleolithic (50,000 BCE)**



Artists conception of communal living in Shanidar cave at the end of the fourth glacial advance (ca. 10,000 BCE)

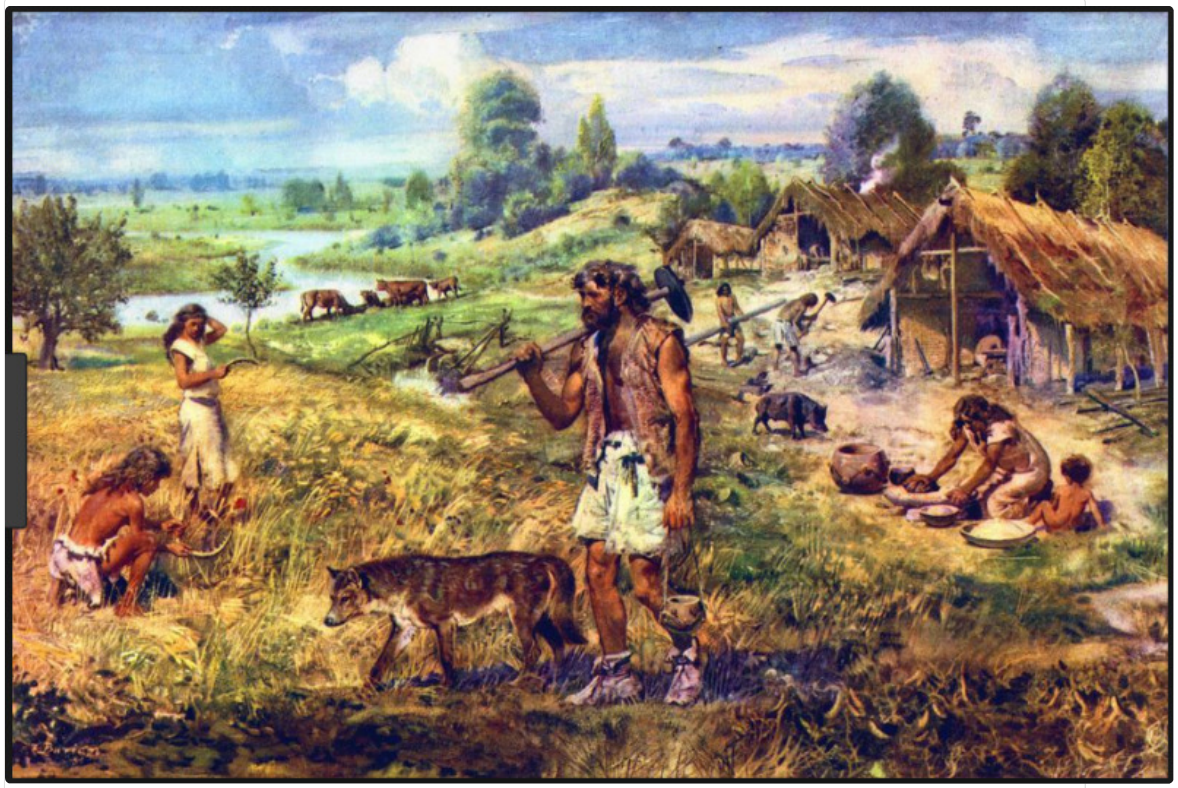
- **Mesolithic (18,000 BCE)** – after the last (fourth) ice age, transition between hunger-gatherer to increasingly sedentary subsistence mode. **Gobekli Tepe (9130 BCE)** on the Anatolian/Syrian border is the earliest known temple site and is unique in that there is no evidence of sedentary living associated with it, or cultivation of grains. The limestone pillars weigh approx. 10-50 metric tons and would have required atleast 500 adults to move and place. Starting to experiment with taming nature: wild cereal cultivation, domestic of dog, domestication of other animals (sheep, goats), mix of semi-settled and

nomadic herders (e.g. **Kebaran culture** in the Levant from 18,000 BCE, **Natufian culture** in the Levant from 12,500 BCE, first evidence of bread 14,500 BCE and beer 11,000 BCE)



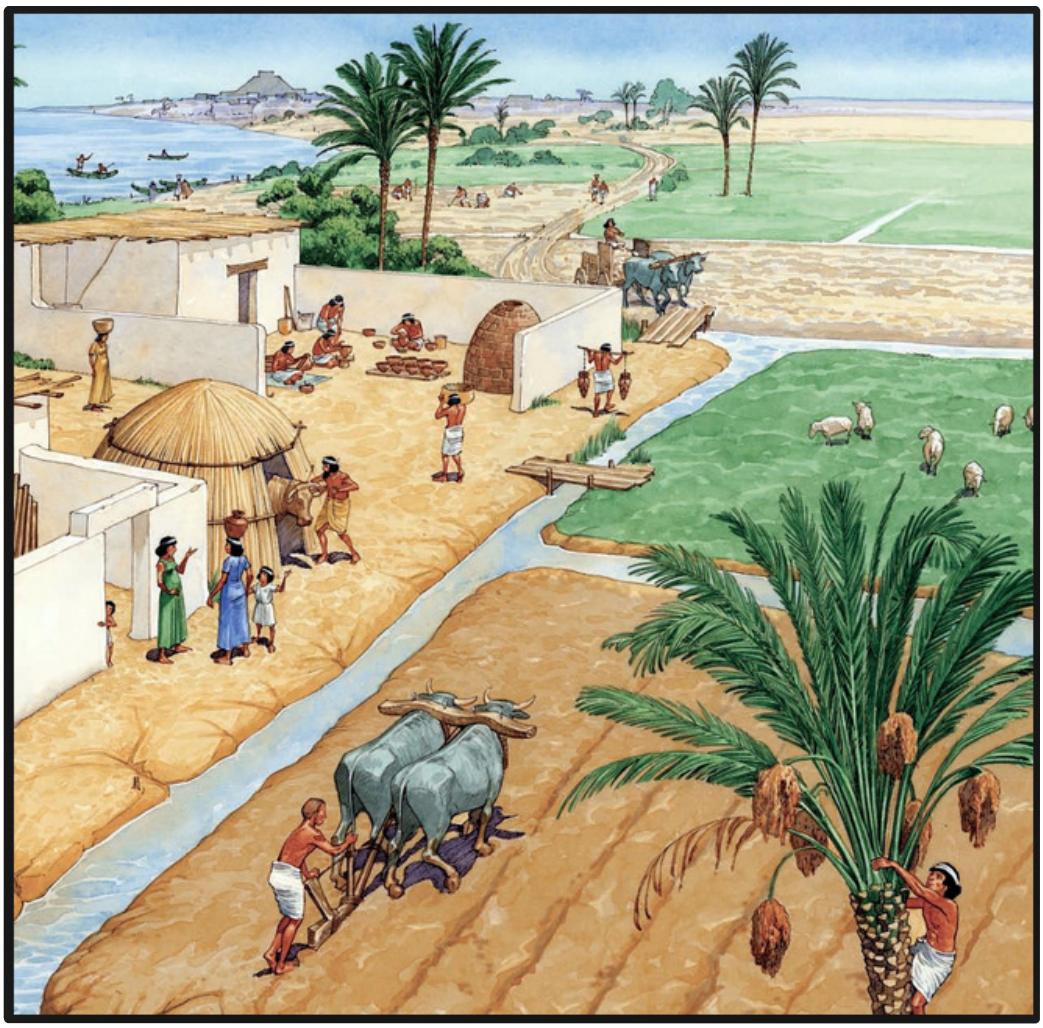
Artists conception of Mesolithic life, combination of hunter and gatherer lifestyle.

- **Neolithic (10,000 BCE)** – culture forming around settlements, herding, farming, but also transhumance seasonal migration between lowlands and highlands. Type site: **Jarmo (7090BCE)**, an agricultural community of 150 people, in the foothills of the Zagros mountains

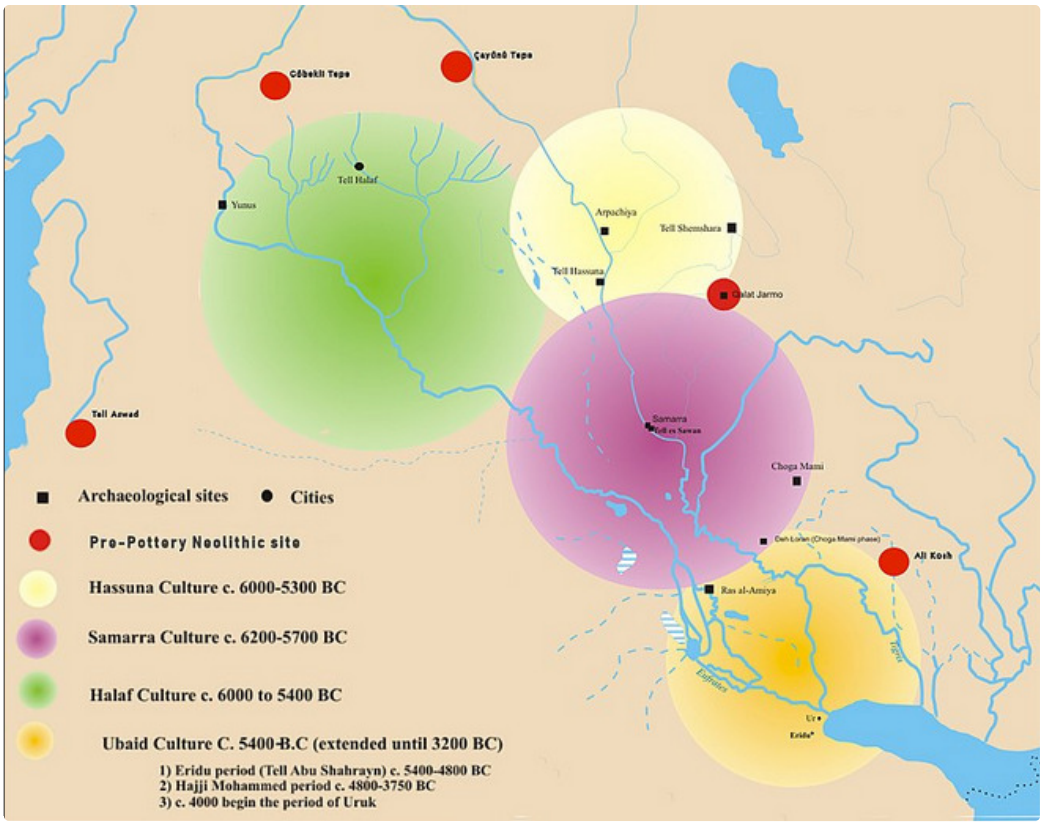


Artist conception of Neolithic lifestyle.

- **Ubaid (6500-3800 BCE)/Chalcolithic (4500 BCE)** – transition to permanent unwalled settlements with specialized craftspeople (potters, weavers, metalworkers), cultivation of grain under arid conditions through the use of irrigation canals (some up to 5km long) requiring large collective labor efforts, the growth of an extensive trade netowrk, and the building of temples. First known settlement in S.Mesopotamia is **Tell el-Ouelli (Ubaid 0)** (6500 BCE-5400 BCE), **4km SE of Larsa**, 25km SE of Uruk. Next is **Eridu (Ubaid 1)** up to 4,000 residents in 20-25 hectares, irrigation agriculture, limited use of copper metal tools, expansion of art and aesthetics, and the beginnings of stratification of society, professional specialization, and the clustering of villages around centers



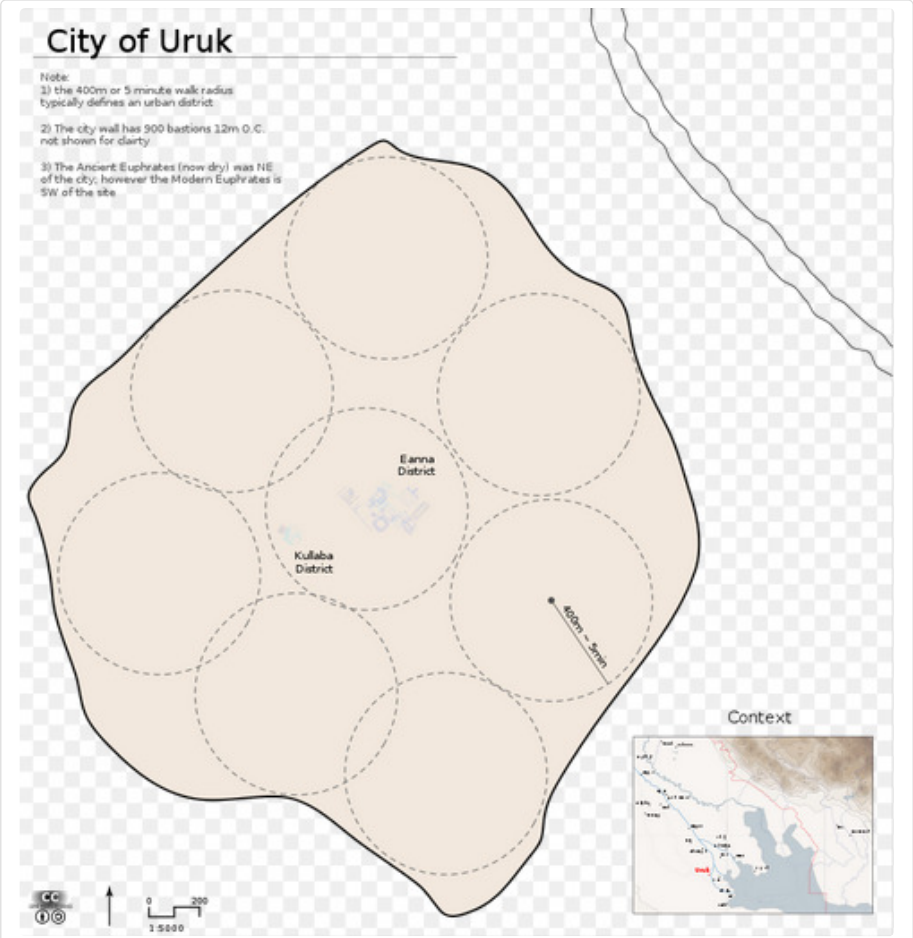
Artists conception of Ubaid life (unwalled settlements, communal labor, irrigation agriculture, copper supplementing stone and wood tools)



Ubaid period cultures, c.6000 BCE onwards.

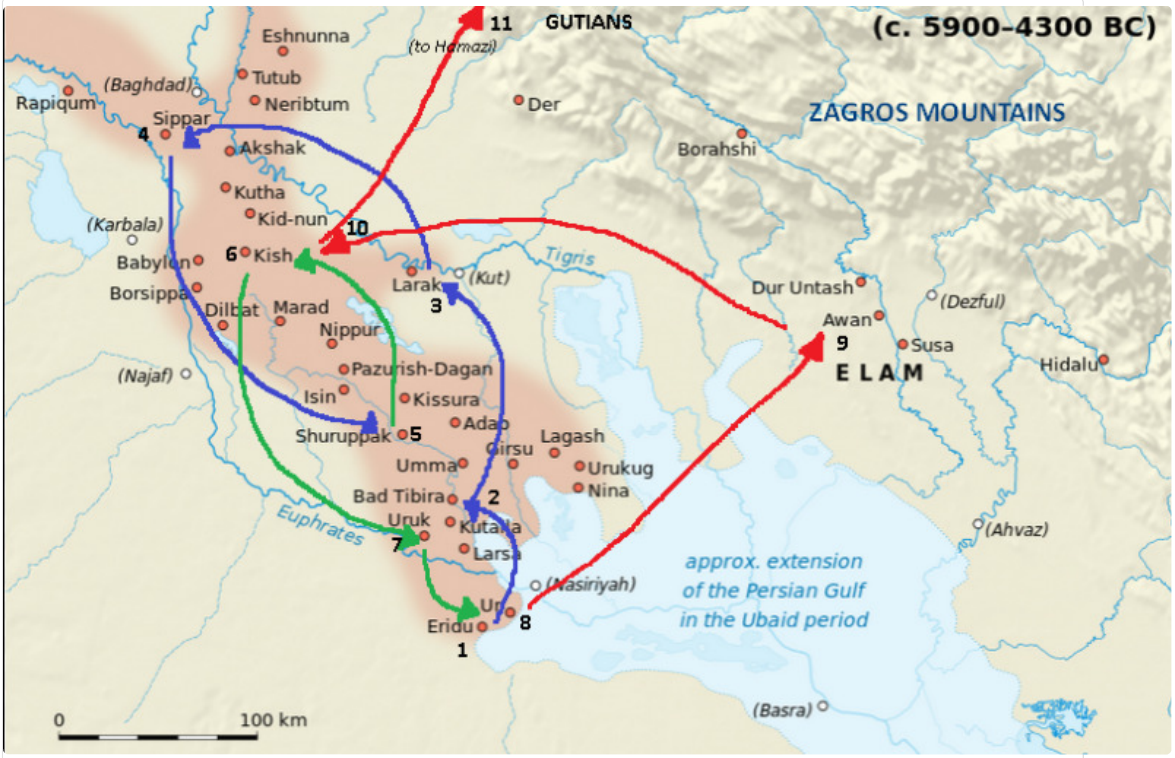
- **City-State Period (4,000 BCE-2,900 BCE)** – early bronze age, expansion of settlement size to large cities with walls (Uruk, Ur, Susa), **with up to 50,000 residents in 6 km2 (Uruk c.2900BCE)**, hierarchical

society with an established elite (temples and lords), warrior class, slavery, long distance trade, large surpluses and the controlled use of labor for prestige buildings – emergence of writing, the state, arithmetic, ancient book-keeping. **Uruk city** (founded in Eridu Ubaid 1 period 5,000 BCE onwards) originated as two separate temple sites to Innana and An (Kullaba district).*See **FAQ1** for discussion of **middle-chronology dating** of Mesopotamian events from Early Dynastic onward*



Uruk City, founded in Ubaid 1 (Eridu) period 5,000 BCE as two temple sites, Eanna (to Inanna) and An, consolidated into a single walled urban site, eventually with an intricate inter-city canal system allowing heavy goods (wood, stone, etc.) to be brought into the city by boat from outposts, colonies, and distant trading partners (Venice in the desert)

- Early Dynastic Sumer (c.2900 BCE) – the establishment of royal lineages and cementing of power and authority, economic and **military rivalry between city-states**.(cf. The Sumerian King List [**Jacobsen/1939**]) The Sumerian King List chronology is: **Eridu**, Bad-Tibira, Larak, Sippar, Shurrupak, The Flood (c.2900BCE), **Kish I** (with 23 kings who ruled after the Flood, including 12th king Etana of the Etana myth, and the 21st king who was the first **archaeologically attested** ruler **Enmebaragesi (c.2600 BCE)**, who built the first temple at Nippur to Enlil establishing **Nippur** as the holy city of Sumer, controlled **Eshnunna**, and conquered **Elam**), **Eanna/Uruk I** (built by Enmerkar who conquered **Aratta**, Gilgamesh), **Ur I**, **Awan (Elam)**, Kish II, **Hamazi (Guti)**, **Uruk II** (contemporary with **Lagash**, whose rulers Ur-Nanshe, Eannatum, Entemena, are, interestingly, not in the Sumerian King List) , Ur II, **Adab**, Maeri, **Kish III**, **Akshak**, Kish IV, Uruk III, **Akkad**.



First 11 cities of the Sumerian King List – from Eridu to Hamazi (Source: Jacobsen/1939)


The Weld-Blundell Prism is the most complete version of the Sumerian King List. Currently housed at the Ashmolean Museum in London. (Source: **Ashmolean Museum**)

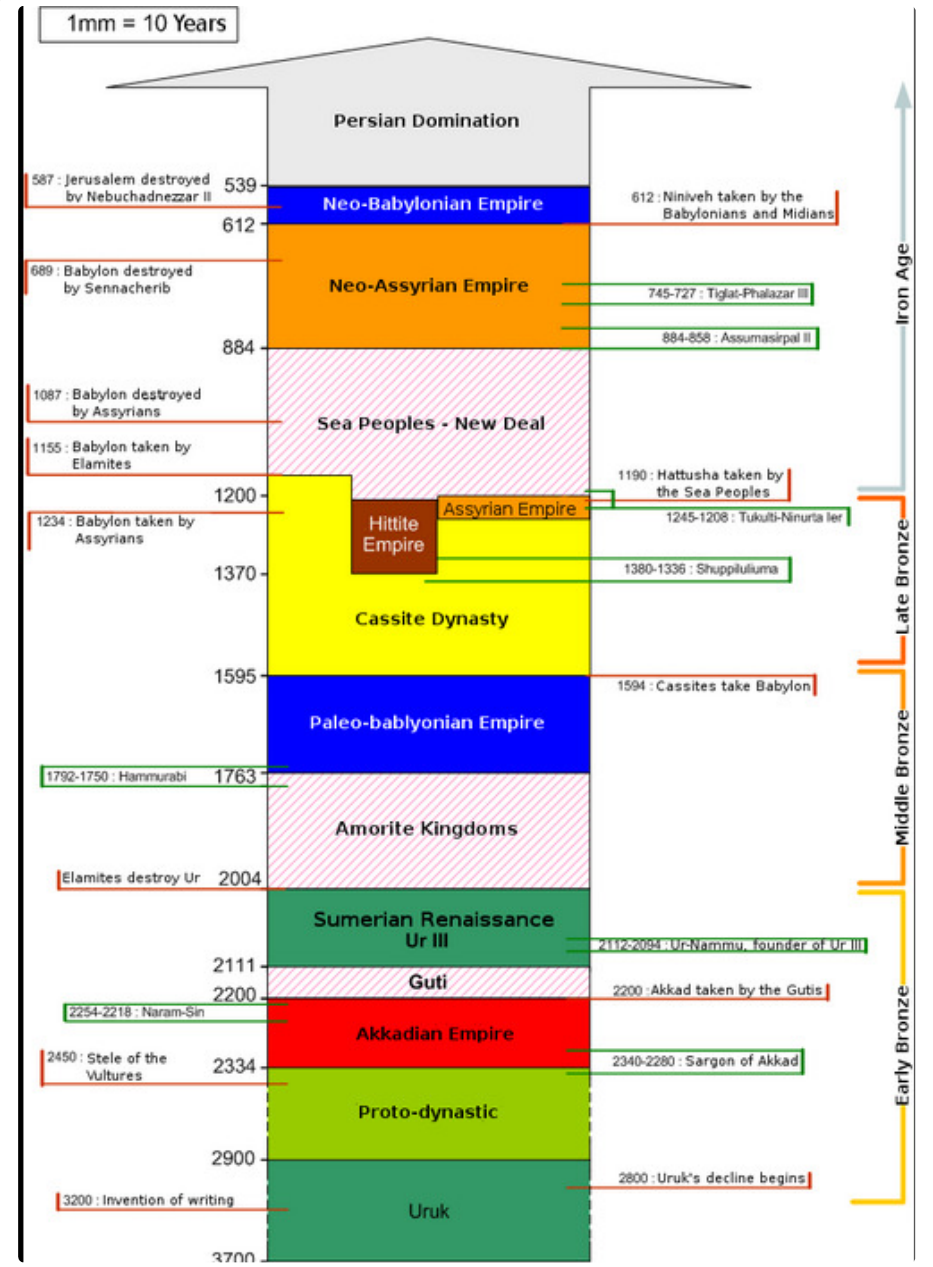
- Age of Empire – unification of the entire region under a single hegemonic ruler, e.g. Sargon of Akkad (cf. [**Jacobsen/1939**], p.111), Ur-Nammu and Shulgi of Ur III (cf. ibid. p.123), and **Hammurabi** of Babylon. The Sumerian King List chronology is: Akkad, Uruk IV, **Gutium**, Uruk V, **Ur III**, **Isin**, where it ends. Subsequently we know it continued: **Isin**, **Larsa**, Babylon.



Cover illustration from The Sumerian King List (Source: **Jacobsen/1939**)



Near East c. 1700 BCE. This is approx. the time of Hammurapi's unification of Mesopotamia centered at Babylon, following the fall of Ur III, the rise of Isin and its subsequent fall to Larsa. Ultimately, the Hittites would sack Babylon and retreat allowing the Kassites (from the Mittani Empire) to rule.



The Cuneiform Empires of the Near East – from Uruk (3700 BCE) through to the fall of Assyria and the start of the Achaemenid (Persian) empire of Darius the Great.

Sumerian Literature

- **Sumerian King List** from the **Weld-Blundell Prism**
- **Enmerkar (of Uruk I) and the Lord of Aratta**
- **Enmerkar and En-suhgir-anna**
- **Lugalbanda in the mountain cave**
- **Lugalbanda and the Anzu bird**
- **Epic of Gilgamesh**: Gilgamesh & Humbaba (Huwawa) (**Version A**, **Version B**), **Gilgamesh and the Bull of Heaven**, **Gilgamesh and Aga** (of Kish I), **Gilgamesh, Enkidu, and the nether world**, **The death of Gilgamesh**
- The Seven Disputations: **Debate between Winter and Summer**, **Debate between Sheep and Grain**, **Debate between Bird and Fish**, **Debate between Hoe and Plough**, Debate between Tree and Reed, **Debate between Silver and Copper**
- **Dumuzid and Enkimdu (Herdsman and Farmer)** (or Innana Chooses Enkimdu)
- Wisdom Literature: **Proverbs Collection**; **Instructions of Shurrupak** (apparently to Zinsudda, aka Utnapishtim, Atrahasis, Noah, before the flood), **Farmer's Instructions**, **Three Ox Drivers from Adab**
- **Epic of Atrahasis** (aka Noah, Utnapishtim, Zinsuddu)

Appendix 4. Assigning dates to historical Mesopotamian events

Mesopotamian events are typically dated in relative terms referencing the “year events” of respective kings within the respective city (e.g. conquest of Larsa by Hammurapi of Babylon in year 30 of his reign). These are associated with each other via synchronism using historical evidence and year event lists from related cities/rulers (e.g. Hammurapi of Babylon year 30 = Rim-Sin of Larsa year 60). They are finally assigned an absolute chronology using one of are 5 competing chronologies for dating Mesopotamian events: high chronology (HC), middle chronology (MC), middle-low chronology (L-MC), low chronology (LC), and new chronology (NC). 152 years lie between High and New chronologies, providing an uncertainty of almost a century for key events such as the Fall of Babylon to the Hittites under Mursuli (direct descendent of Hattusa).

A new technique using carbon 14 dating of tree rings (dendrochronology) has been able to narrow this uncertainty down to +/- 8 years by demonstrating that the only viable chronologies are the middle (MC) and middle-low chronologies (L-MC), and that both of these are currently compatible with known astronomical evidences. The fall of Babylon is now established as between 1587-1595 BCE. [Manning/2016]

Other key dates in the middle chronology (Ref: [Fitzgerald/2002])

- Shulgi of Ur III, 2091 BCE
- Rise of Larsa, 1941 BCE (Zabaia, fourth named king of Larsa).
- Fall of Isin to Larsa, 1794 BCE (Rim-Sin I of Larsa conquering Damiq-ilisu of ISin in Year 29 (rim-sin) and Year 23 (damiq-ilisu0
- Hammurapi takes throne of Babylon, 1792 BCE (year 31 of rim-sin of Larsa) (Old Babylonian period)
- Fall of Larsa to Babylon, 1763 BCE (year 60 of rim-sin of Larsa, year 30 of Hammurapi)
- Fall of Babylon to Hittites, 1595 BCE. Start of Kassite dynasty (from Zagros mountains) (End of Old Babylonian period, Start of Middle Babylonian period)
- Fall of Kassite Babylon, 1155 BCE.

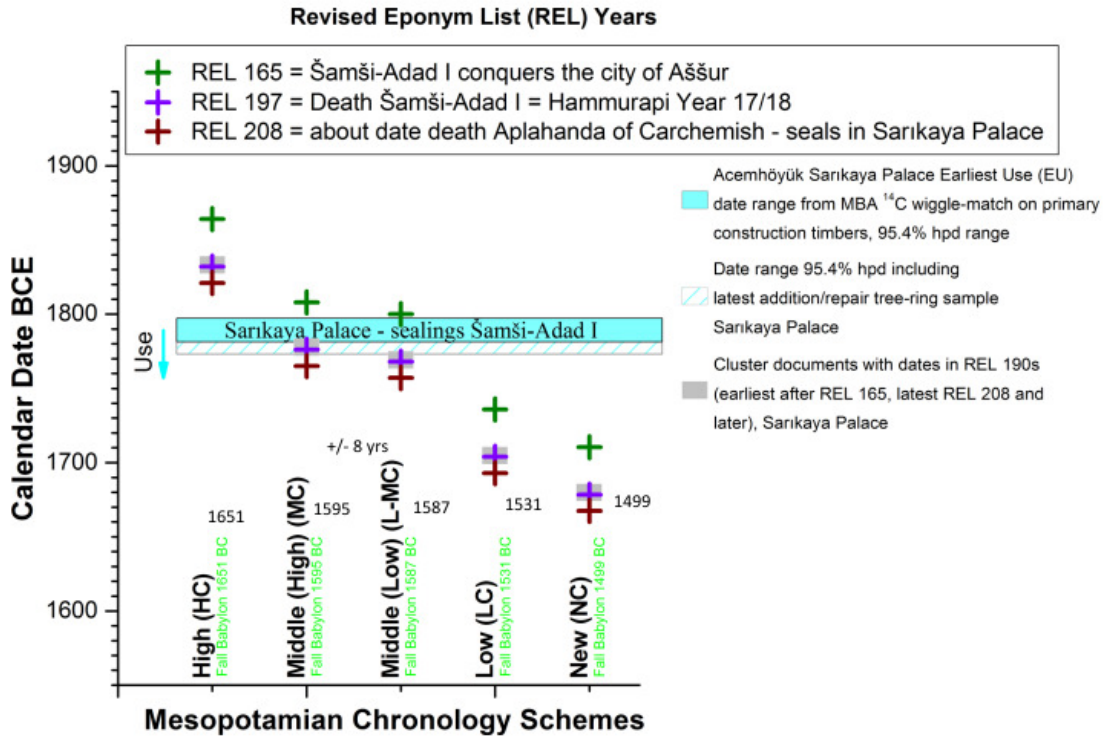


Fig 8. Comparison of the proposed calendar dates BCE from some of the main Mesopotamian Historical Chronologies [5, 9–19, 73] for four key REL dates [5] versus the dendro-^{14}C-wigglematch date ranges (95.4% probability range) for the likely Earliest Use (EU) of the Sarıkaya Palace at Acemhöyük [23] as +1 year from latest primary construction timber with bark/waney edge (data from Table 2, Model 8a, Table 3), and including the subsequent dendrochronologically known repairs/additions to the building [23]. The known lifetime of the Sarıkaya Palace runs from the time of Šamši-Adad I (with nothing earlier) through sealings of Aplahanda of Carchemish who died around REL 208. The dates indicated are: (i) REL 165 (Šamši-Adad I conquers Aššur); (ii) REL 197 (death of Šamši-Adad I) [5]; (iii) REL 190s which represent a cluster of dates on documents at the Sarıkaya Palace [29]; and (iv) REL 208. The respective calendar dates BCE for the Fall of Babylon (sacked by the Hittite king Muršili I) according to the five main Mesopotamian Historical Chronologies [5, 9–12, 15–19, 73] are given in green text also (note: the chronology of Mebert [19], 8-years later than the Low Chronology, is not shown).

C14 dating of tree rings has improved absolute dating to within +/- 8 years, and validated Middle (MC) and Middle-Low (L-MC) chronologies. Source: [Manning/2016]

Appendix 5: Culture in the Near East: From Mesolithic (after the last ice age) to Neolithic (rise of sedentism)

Middle Stone Age (Mesolithic): After the Last Ice Age but before Sedentism

References: Charvat/2002, Nissen/1988

By 14kya, the Paleolithic era is ending with the last ice age giving way to the Younger Dryas inter-glacial period (lasting to the present).

The change in climate and habitat triggers the start of the Mesolithic epoch in the transition from Paleolithic (nomadic hunter-gatherer) to Neolithic (settled farming).

As the last ice age receded around 12kya, the environment began to yield much more plentiful food sources for hunter gatherers.

In the Mediterranean and Near East, wild grasses and cereals (wild barley, einkhorn and emmer wheat) increased, accompanied by lentils and various pulses.

Hunting moved from indiscriminate killing of a wide range of animals to a focus on a few species, particularly wild sheep, wild goats, and onager (wild donkey), supplemented less intensely by deer, wild boar, wild cattle, hare, wolf, fox, various birds, and other small mammals.

Gathering was turtles, hedgehogs, snails, other molluscs, and plant food.

For tools, they processed stone, bone (awls, knife handles, etc.), wood. To form axes, they used bitumin, a naturally occurring sticky tar, to attach stone or obsidian (black volcanic glass) blades into notched handles.

Stone tools were typically made of chert or flint, obsidian or quartz (**hardest material**)

> Abundance spurred curiousity, new resources were opened (a variety of stones, raw copper, bitumen) and new technologies such as grinding and polishing stones and even the first steps in chemical production (lime plasters), were introduced." (**Charvat 2002**:10)

Even in this period of plenty, there is evidence of violence between human beings.

By 11kya, they started to settle down in semi-fixed homes and experiment with mixed mode living — hunting, gathering, herding, and with wild crops.

By 10kya, man had domesticated other animals, not unsurprisingly first sheep and goats, given the heightened contact through hunting, and then cattle. By 9,500 ya, evidence exists of domesticated pigs.

By this point, housing was kept scrupulously clean, with lime plaster or clay interior walls, lowered floors, spiritual or ritual objects – statuettes, grave goods, body ornaments (necklaces, bracelets, rings). (**Charvat 2002**: 13)

> "The essential characteristics of all human communities up to recent time—economic specialization, social differentiation and complex spiritual reflection of the visible world—may be documented in this period of time." (**Chavrat 2002**:15)

New Stone Age (Neolithic) and Rise of Sedentism

Now we enter the Neolithic period, in the Near East this is from 9500 BCE onwards.

In agriculture there is experimentation with emmer wheat and einkhorn wheat, also cultivated peas, lentils, six-row bread wheat, oats, rye, linseed, and flax. They gathered wild cereals and also pistachio nuts from the highland woods.

Some of the dwellings (e.g. at Umm Dabahiyah) now have "fresco paintings geometrical patterns and figural scenes (an onager frieze, a hunting scene)" (Charvat 2002:19) — fresco paintings are with colored powders applied to fresh plaster so that upon drying, the painting is an integral part of the wall.

There is pottery, and ornamental decorations on pottery, houses with complex structures, including stairways for roof access, kilns for firing pottery, textiles and woven baskets.

In agriculture there is artificial irrigation leading to larger crop yields (sites Choga Mami and Tell es-Sawwan inter alia). There is also clear evidence that Neolithic peoples were experimenters, cross-breeding cereal grasses to obtain domesticated variations that are in use to this day (four-row emmer wheat, six-row variations, with non-shattering stems in order to preserve the grains when harvesting). Charvat 2002:30

This was their main achievement – the advance and experimentation in securing additional food sources and improving and perfecting these through genetic interventions.

Dogs were used to assist in both shepherding and hunting. As shepherding added more species (goats, sheep, cattle, pigs), hunting targets changed away from wild goats to gazelle and onagers, presumably because wild goats would be added to the herd. Gazelles and onagers are harder to hunt, requiring the coordination of numerous hunters with a single purpose. They also consumed more fish, mussels, turtles, and crabs.

The settlements consisted of several houses, in some cases (Tell es-Sawwan) with a fortification ditch (3m

deep, 2.5m wide) surrounding the houses, together with a rampart with buttresses (reinforced defensive walls). Houses were built of clay bricks, sometimes formed in molds, and the floors bore an occasional coating of bitumen (tar) or gypsum, otherwise reed mats, or stamped earth. Some of the village streets were paved.

By Neolithic time, civilization was complete — societies had structure, religion, economic specialization, surplus food, art, and community.

Planting and harvesting of grain was in place by 6900-6000 BCE.

> "The Neolithic food-producing economy was no fun … human remains at the site show evidence of physically demanding work, including collapse of neck vertebrae due to carrying heavy loads on the head (remember that the wagon would not be invented until the Chalcolithic period [from 4000 BCE on]." The digging, threshing, grinding, harvesting, irrigation, water carrying, load carrying — all of this was human labor. (Charvat 2002:32)

One wonders if in the Neolithic times ideas of slavery led to raids on nearby settlements in order to coerce others to carry out food-producing work.

Lifecycle of lambs and sheep: "Lambs are born around Christmas (for confirmation by the Near Eastern data see Wright, Miller and Redding 1980, 271; Wright, Redding and Pollock 1989, 108–109; Hruška 1995, esp. pp. 82–83) and in May they are usually grown enough to walk even over heavy ground and to be weaned so that sheep can be milked from that time on. In May the shepherds with their herds usually ascend the summer pastures whereupon the sheep are sheared and new wool employed to settle all accounts, debts and obligations that the shepherds or their masters might have incurred before, the season of cheesemaking following in the months of June and July. (Charvat 2002:39)

There is some evidence that Neolithic cultures were migratory, moving seasonally between lowlands (winter) and highlands (summer), and taking advantage of whatever combination of subsistence methods worked in each circumstance. So there was agriculture, herding, hunting, gathering, but looks like there may have been migrations twice a year, (Charvat 2002:47) I.e. the sites were permanent but the people in them were not (Charvat 2002:40) Non-nomadic domesticates are the pig, which cannot travel long distance. Another sign is larger cemeteries indicating territorialization of human communities. (Charvat 2002:39)

Appendix 6: Ancient Stone Age (Paleolithic) and the Four Ice Ages

The Paleolithic epoch begins with the appearance of **the earliest stone flake tools 3.3mya near Lake Turkana in Kenya**.

Australopithecine disappeared from the fossil record about 1mya, replaced by Homo genera of the Hominidae family. But for a time there is overlap between co-existing distinct human species.

From 2.5 mya to 1.3mya we have Homo habilis (`handy man'), with more sophisticated stone tools near Oldowan in Tanzania.

From 1.9 mya to 400k ya we have Homo erectus.

Note that, during this period from 1.9mya to 1mya, for almost 900,000 years, there were at least 3 species of humans overlapping in the same range in eastern and north-eastern Africa (australopithecus, homo habilis, and homo erectus).

Three more species of humans fill out the story: Homo sapiens, appearing about 600kya, Homo sapiens

neanderthalis appearing about 130kya, and Homo sapiens sapiens (anatomically modern man) appearing about 100kya.

Throughout this period, waves of human ancestral migration leave Africa and populate the earth. Homo erectus remains of 1mya are found in the Jordan valley (Ubaidya site).

By 700kya, Homo erectus man has learned to control fire, potentially accelerating the evolution of man to homo sapiens as a more protein rich diet could be consumed through cooking, leading to brain expansion. This theory is contested by the discovery of small brained human ancestors who were every bit as smart as the larger brained successors. (citation)

By 100kya, most of the Earth has been populated by hominid ancestors.

The last glacial period began 110kya and lasted to 11,700 years ago, or to 9700 BCE.

By about 50kya, a wave of modern humans (homo sapiens) left Africa and moved through the Fertile Crescent (**Map of Pre-historic sites**)

In this early period (48kya-35kya), we have both Neanderthal and Homo Sapiens occupying the same region, with the Neanderthal skeletons of Shanidar Cave in the Zagros mountains providing possibly the earliest evidence of human assault on Neanderthals. The Skeletons of Shanidar Cave, in the Zagros mountains of Kurdistan in northern Iraq, are Neanderthals from 65kya to 35kya. 10 nearly complete Neanderthal skeletons provide a remarkable picture. Neaderthals cared for their wounded and buried their loved ones in graves. There is evidence of murder of `Shanidar Three' by a low-mass, low-kinetic energy projectile weapon, either by fellow Neaderthals, or by projectile-carrying homo sapiens who had entered the region. In contrast with the killing projectile, Neanderthals used heavy huntings spears thrust with great force at close range into their prey (large mammals). Humans had mastered lighter projectile spears, throwing these deftly and with some accuracy from further away. (**Article 1**, **Wikipedia: Shanidar Cave**, **Article 2**, **Article 3**)

By this time 14mya man has mastered the bow and arrow, and we have evidence of domesticated dog living amongst men (**evidence at Palegawra site (1975) in the Zagros mountains**. How did this come about? Russian and US scientists have **identified a gene (SorCS1) in foxes that leads to exaggerated friendliness and tameness behaviour (2018)**. A tameness gene may have made a few wolves seek human affection. If female, this may have led to pups with the same gene, and over the course of hundreds of generations of animals, to dogs.

Appendix 7: Birth of the Universe to the Origins of Man

The Universe was formed 13.8 billion years ago (bya), +/- 21 million years (a measurement error of 15 years per million). **Within the first second**, the universe underwent a massive expansion accompanied by hypercooling and the formation of the fundamental forces and particles. **Over the next 20 minutes**, light formed (photons) and matter was created (mostly hydrogen and helium). The universe was a super-hot glowing fog (10^9 Kelvin K, or 10 billion degrees Celsius). It would take 370,000 years for the fog to dissipate, the universe to become transparent, and to cool a million-fold to 3000 K (2730* C). **Temperatures of 300K, suitable for liquid water and life** (the habitable era of the universe) were reached 10 million years after the big bang.

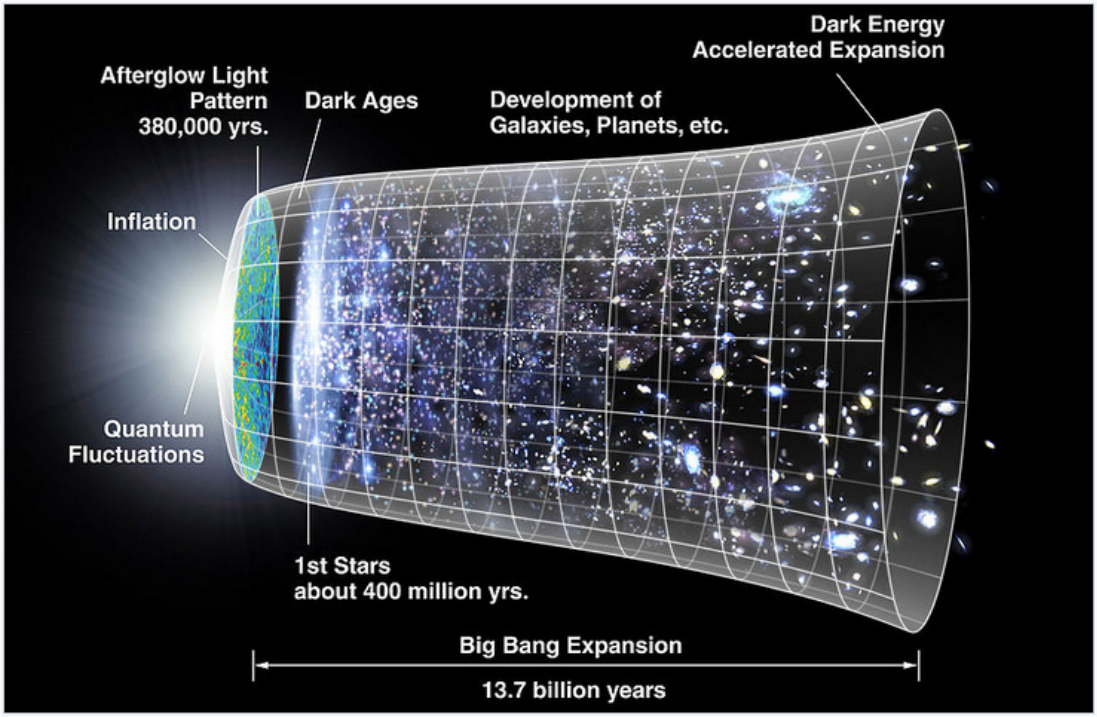


Diagram of Evolution of the universe from the Big Bang (left) to the present

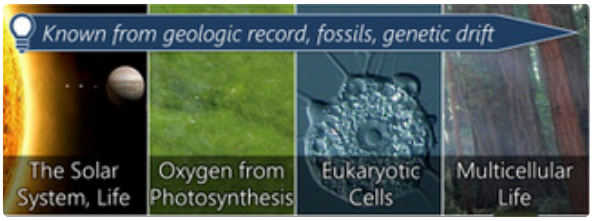
Birth of Universe 13.8 billion years ago (bya). Source: Wikipedia: **Timeline of epochs in cosmology**

Star formation began after 100 million years, ending the so-called **"Cosmic Dark Ages"**. Stars began nuclear fusion reactions from 300 million years to form the heavier elements. The first galaxies appeared at 400 million years. Our Milky Way galaxy began to form after 700 million years, and would take the next 4.5 billion years to evolve until it acquired its spiral arms through galaxy collision (8.7 billion years ago bya). It would be another 4 billion years till our solar system formed.

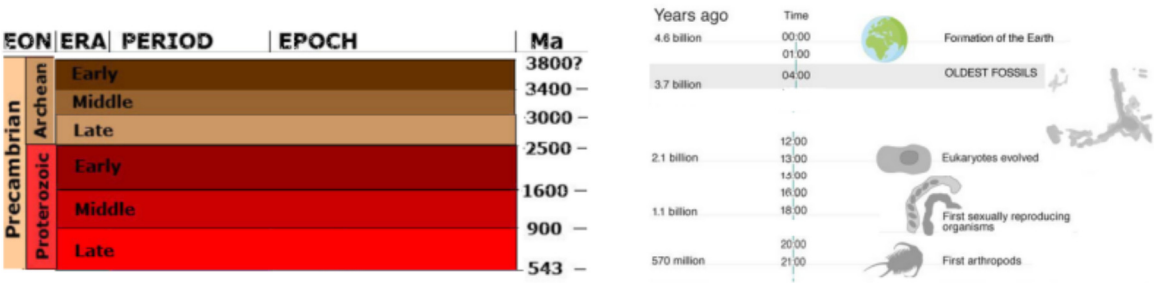


The First 9 Billion Years. From the Birth of the Universe (13.8 billion years ago) to the formation of the Milky Way (760 million years after birth) to the start of the Milky Way spiral (5 billion years), to the formation of our solar system (9.2 billion years, 4.6 billion years ago) Source: Wikipedia **Matter Era of Cosmology**

Our solar system, including the Sun, the planets, and the Earth-Moon system, formed over a **100 million year period** between 4.6 and 4.5 billion years ago (bya). **What happened next on Earth** is still controversial, including by when the **molten earth cooled**, formed a crust, and contained surface liquid water. Also controversial is whether and to what extent a **Late Heavy Bombardment period** occurred during which large numbers of asteroids from the solar system's outer belt were pulled into the inner solar system and collided with many of its bodies, including the moon, and Earth itself. The **first evidence of life on Earth** follows immediately after this, with single-celled organisms (bacteria) first appearing in the fossil record 3.8-3.7 bya (Early Archaen Era). Over the next 3 billion years life evolved slowly with the **first exchanges of genetic material amongst prokaryotes occuring 2bya**, **first multi-celled organisms** appearing 2.7 – 1.6 bya (algae, amoebas, mold, fungus), and first sexually reproducing simple multi-cellular organisms appearing 1.1 bya (Middle Proterozoic era).

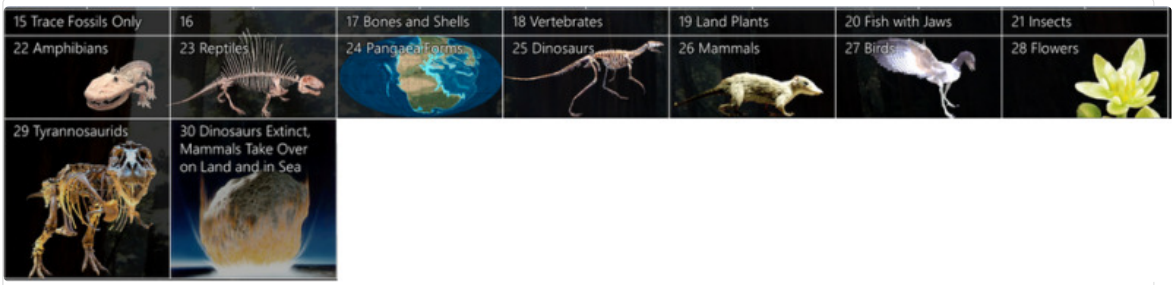


First 2 billion years on earth: from Earth formation (4.5 bya) to multi-celled organisms (2.6 bya)

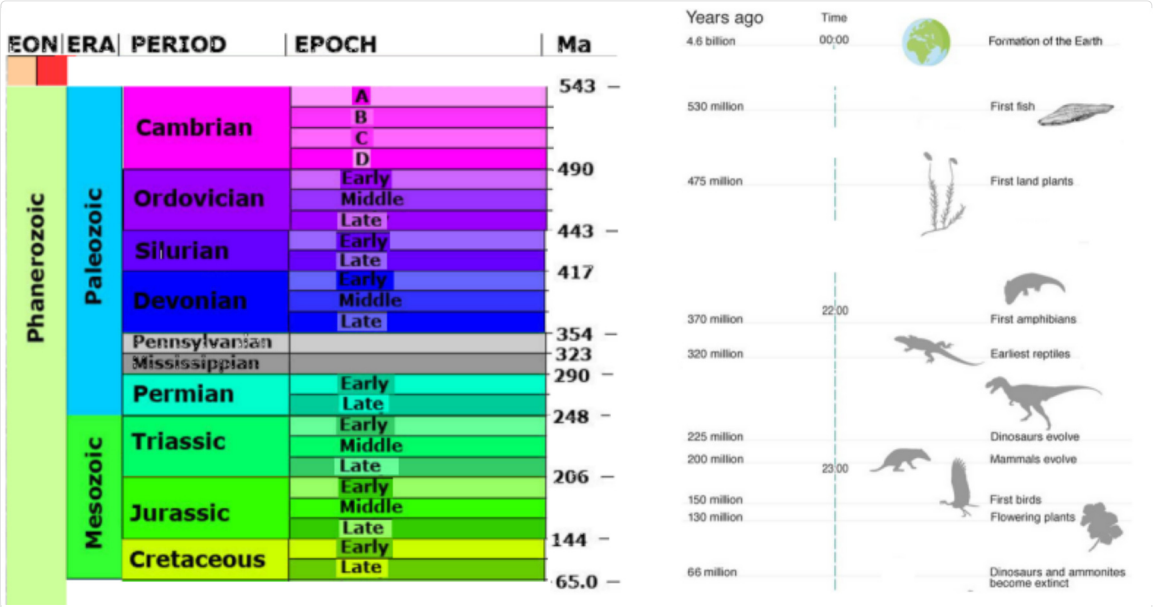


The first 4 billion years on Earth (4.6 bya): evidence of earliest life found 3.8bya, reproducing multi-celled organisms by 1.1bya, **the first animals (sponges) around 670mya**, and the **first arthropods (invertebrates)** by 570-555mya. Sources: **GeologyCafe.com** and Unknown

The next 500 million year period is fascinating, from the Cambrian explosion (500 mya) when life diversity accelerates rapidly to include fish, plants, reptiles, dinausaurs, birds, mammals, through to the cataclysmic event that ended the dinosaurs, triggered that cooling of the tropical earth and the rise of modern habitat and dominance of mammals. The cause of the acceleration in diversity is controversial. A **new paper (2018)** presents a provocative thesis – that the Cambrian explosion may have been triggered by the insertion of extra-terrestrial DNA into the earthly mix, brought in by a meteor or comet. Researches from the past decade have suggested that another major evolutionary event occured in the middle Triassic period, during the so-called **Carnian period around 252mya**. At this time, volcanic explosions are thought to have increased CO2 atmospheric content, warmed the global atmosphere, leading to 1-2 million years of rainfall on what had been previously an arid, dry Earth, and again accelerating diversity (dinosaurs, origin of mammals and many other modern forms of life.)



The Last 500m Years: From the Cambrian explosion to the end of the dinosaurs



Next 500 years, from the Cambrian Explosion (500mya) to the extinction of the dinosaurs (66mya)

In the aftermath of the dinosaurs about 66mya, mammals flourished and primates appeared (13 mya).
By 3.8-3.5mya the earliest bipedal hominids (Austraopithecine) appear in Africa (Tanzania, Kenya, and Ethiopia), and the **stone age (paleolithic period)** begins (see **Appendix 6**).



Last 20m Years: from the appearance of primates (13mya) to the migration of anatomically modern humans out of Africa (c.60,000 years ago)

Download article (PDF)

Bibliography & Further Reading

Mesopotamian Mathematics: From prehistoric metrological tokens to writing and the earliest recorded mathematical practice (3200 BCE and onwards)

1. Denise Schmandt-Besserat, 1977, **An Archaic Recording System and the Origin of Writing**,"; Syro-Mesopotamian Studies I., 1977, pp.31-70; [Besserat/1977]
   *This first publication of her findings builds on prior work of Amiet (1966) on Susa findings, on Oppenheim (1959) on Nuzi findings including an inscribed bulla from 2000-1500BCE, and on Falkenstein (1936) on archaic signs (proto-writing). Subsequent detailed investigations of Besserat's hypothesis have supported the following points (1) sealed bullae containing tokens provide the evidence of the use of tokens for accounting commercial transactions, (2) that this transition from tokens to inscribed bullae provides a key missing link between pre-writing numerical practice, proto-writing, and the proto-cuneiform that followed, (3) that this critical transition happened c.3200 BCE in Uruk (aka Warka) in southern Mesopotamia. The rest of her many claims in subsequent publications have been demolished, in particular the claim that clay tokens were an accounting system in wide use across the Near East. See critical reviews by **Zimansky/1993**, **Englund/1993**, **Englund/1998**, and the use of contextual archaeology to close the case on Besserat's speculations, see masters thesis **Niemi/2016**, and **Bennison/2018***

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